Actuator Noise for the Second Stage
of the Seismic Isolation and Alignment System

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This note describes a simple estimate of the allowed noise of the voltage drivers for the second stage of the Advanced LIGO SEI platform. The allowed voltage noise is a smoothly function with maximum values of:
15 nV/√Hz at 30 Hz and above,
38 nV/√Hz at 10 Hz,
210 nV/√Hz at 1 Hz, and
changing as 1/f below 1 Hz.

Requirement at 10 Hz

At 10 Hz the system is essentially a free body being acted upon by multiple actuators. The system has 6 DOF, we assume x and y are the same, so there are 4 unique degrees of freedom. We need to know the mass, the moments of inertia, the geometry of the actuators, and the table size. Then, we calculate the requirement for the translational degrees of freedom from:

$$allowed\_force = mass * allowed\_displacement * \omega^2$$

Once we know the allowed force, we use the resistance and force constants of the actuator to determine the allowed voltage noise. For the rotational degrees of freedom, we calculate how much translation will be generated at the edge of the table from a given rotation, and how much torque will be generated by the separated force actuators. The table radius is approximately 96 cm.

$$allowed\_torque = moment * \frac{allowed\_displacement}{table\_radius} * \omega^2$$

Actuator Geometry

The actuators are arranged as the corners of an equilateral triangle, 3 are vertical and 3 are tangential, allowing control of all 6 DOF of the platform. We assume the noises of the actuators are independent but have identical spectral amplitudes. We can calculate the how the noises of the various actuators combine to drive the system in the coordinate directions.

vertical translation – 3 actuators in concert against mass,

√3 * noise of 1 actuator = 1.73 * 1 actuator

horizontal translations – 3 actuators, not in-line, against mass.

1 actuator in line + 2 at 60 deg off (or 2 at 30 deg off),

$$\sqrt{\left(\sqrt{3}/2\right)^2 + \left(\sqrt{3}/2\right)^2} = \sqrt{3/2} \cong 1.22$$

(this is the same as sqrt(1^2 + $½^2$ + $½^2$))

rotation about z, 3 actuators in concert against z-moment. 1.73 * actuator radius * 1 actuator,

The vertical actuators are about 70 cm from the central axis.

rotation about x&y, 2 actuators in concert against x or y moment. 1.41* actuator radius * 1 actuator,

The horizontal actuators about 66 cm from the mid-line.

Stage Mass

For the mass, we use numbers from the ASI estimates in the spreadsheet "System_Rigid_Properties_v5.xls". For the fixed mass of stage 2, we use 2172.7 kg. The actual mass will probably vary from 2172.7 kg (BSC 8-1) to 2404.1 kg (BSC2-2).

The average moments for BSC with 295 kg ballast + fixed masses are:

direction	mean	min
Ixx	866	622 kg m^2
Iyy	870	622 kg m^2

Izz 779 612 kg m^2

We use the minimum values of 622 kg m^2.and 612 kg m^2.

Requirements for the Various DOFs

We can now calculate the requirements for the 4 different DOFs.

direct translation in x / force noise per actuator = 1.22/(ω^2 rad/sec * 2172.7 kg) = 1.42e-7 m/N at 10 Hz
direct translation in z / force noise per actuator = 1.73 /(ω^2 rad/sec * 2172.7 kg) = 2.02e-7 m/N at 10 Hz
translation in z from rotation about x&y / force noise per actuator
= 1.41 * .96 m/rad * .70 m/(ω^2 rad/sec * 622 kg m^2) = 3.87e-7 m/N at 10 Hz
translation in x&y from rotation about z / force noise per actuator
= 1.73 * .96 m/rad * .66 m/(ω^2 rad/sec * 612 kg m^2) = 4.54e-7 m/N at 10 Hz

The motion requirement at 10 Hz is 2e-13 m/√Hz.
To meet this noise level, with no benefit from active control, with the worst case motion from force, (motion and the table edge from rotation about z)
force noise per actuator = 2e-13 m/√Hz / 4.54e-7 m/N = 4.41e-7 N/√Hz

The specs on the actuators coming from PSI are
2.4 amps gives 10 lbs of force, so the force constant is:
44.5 N/2.4 amp = 18.5 N/amp
With this force constant, the equivalent current noise is 2.38e-8 amp/√Hz
The voice coil drive is a voltage drive, and the coil presents a resistive load of 4.7 ohm. This means the equivalent voltage noise is 1.12e-7 V/√Hz, i.e. 112 nV/√Hz.

Margin and Suppression.

The *equivalent noise* is the noise which will drive the system at the level described in the SEI requirements document. We recommend a factor of 3 margin on the 10 Hz actuator noise requirement. This means that the open loop measurements of sensor performance won't be impacted much by actuator noise, and when the loops are closed with the predicted upper unity gain frequency of 30 Hz, an additional factor of 3 in suppression from the loop will put the amplitude of the actuator noise at 10% of the performance requirement.

The 10 Hz voltage noise requirement is 37 nV/rtHz = 1/3 * 112 nV/rtHz.

Requirement at 30 Hz

All of the scaling arguments at 10 Hz apply at 30 Hz as well, except the loop gain and the final platform noise requirements are different. The platform motion requirement is 3e-14 m/√Hz, and the open loop gain will be nearly 1.
Translation coupling from x&y rotation / force noise per actuator = 5.04e-8 m/N at 30 Hz
Equivalent force noise per actuator = 5.95e-7 N/√Hz
Equivalent current noise per actuator = 3.22e-8 amp/√Hz
Equivalent voltage noise per actuator = 1.51e-7 V/√Hz
desired margin = 10

The 30 Hz voltage noise requirement is 15 nV/rtHz = 1/10 * 151 nV/rtHz.

Requirement at 1 Hz

At 1 Hz, the system is not a free body, instead the compliance is dominated by the offload springs. Thus the translation requirement is determined by:

$$allowed\_force = allowed\_displacement * k$$

For the rotational DOFs, we again determine the displacement at the table edge based on rotation from the torque generated by separated force actuators

$$allowed\_torque = \frac{allowed\_displacement}{table\_radius} * \mathrm{K}$$

Spring Stiffness
The stiffness of the springs is also given by the ASI document, "System_Rigid_Properties_v5.xls".
For the BSC-295, the stiffness are given as:

blade k = 2.77e5 N/m,
rod k = 2.36e5 N/m,
rod radius = 0.699 meters

From these numbers we can calculate the 4 different DOF stiffnesses.

horizontal stiffness = 3*rod k = 7.08e5 N/m
vertical stiffness = 3*blade k = 8.31e5 N/m
rotational stiffness about x&y =
$2 * \sqrt{3}/2 * \text{rod radius}^2 * \text{blade } k$ = 2.34e5 N*m/rad
rotational stiffness about z =
$3 * \text{rod radius}^2 * \text{rod } k$ = 3.46e5 N*m/rad

With the actuator geometry given in the 10 Hz section, we can calculate the coupling from a single actuator's worth of noise to motion at the table edge, as x = F/k for translation, or x = angle*radius = F*actuator_radius*table_radius/K for the rotational degrees of freedom.

Coupling Calculation
direct translation in x / force noise per actuator = 1.22/7.08e5 N/m = 1.72e-6 m/N
direct translation in z / force noise per actuator = 1.73 /8.31e5 N/m = 2.08e-6 m/N
translation in z from rotation about x&y / force noise per actuator
= 1.41 * .96 m/rad * .70 m/(2.34e5 N-m/rad) = 4.05e-6 m/N
translation in x&y from rotation about z / force noise per actuator
= 1.73 * .96 m/rad * .66 m/(3.46e5 N-m/rad) = 3.17e-6 m/N

At this frequency, the worst case is rotation about x, with a coupling of 4.05e-6 m/N
The motion requirement[*]T at 1 Hz is 1e-11 m/√Hz.
The equivalent force is 2.47e-6 N/√Hz = 1e-11 m/√Hz / 4.05e-6 m/N.
For the PSI actuators, the force constant is 18.5 N/amp, so the equivalent current noise is 1.33e-7 amp/√Hz, or 630 nV/√Hz with 4.7 ohms.
This assumes open loop motion equivalent to the final motion requirement. We again apply a margin of 3.
The 1 Hz voltage noise requirement is 210 nV/√Hz = 1/3 * 630 nV/√Hz.

Below 1 Hz, the noise may rise as 1/f.
At 0.2 Hz, the motion requirement is easily met if noise is ~1/f.
Below 1 Hz, the biggest problem will be vertical noise generating tilt-horizontal coupling.
The coupling from actuator noise to tilt about x&y is
tilt/ force noise per actuator = 1.41 * .70 m/(2.34e5 N-m/rad) = 4.22e-6 rad/N.
The detection of tilt as horizontal motion is g/ω^2. For the perceived open loop horizontal translation at 30 mHz to be 2e-7 m/√Hz, the force noise will be

2e-7 m/√Hz = g/ω^2 * tilt/force noise per actuator * force noise per actuator.
force noise per actuator = 2e-7 * (ω^2/g) / 4.22e-6 = 1.71e-4 N/√Hz

The loop gain in this region is large. It would not be surprising for it to be greater that 1e5, so it is difficult to imagine that this will ever be a problem. However, if we proceed, we get a current noise equivalent of 9.24e-6 amp/√Hz, and a voltage noise of 43 μV/√Hz at 30 mHz. If we apply the customary factor of 3, this becomes 14 μV/√Hz. If the noise is rising as 1/f below 1 Hz, it will "only" be 6.3 μV/√Hz at this frequency, so the 1/f scaling is acceptable.

[*] The requirement is actually an rms requirement between 1 and 10 Hz, which this will meet