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# Diagonalization of the Input Matrix of the Suspension System

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# Abstract

We developed a new and reliable method for diagonalizing the input sensing matrices of the digital suspension controls system at the Caltech 40m lab. By using this method, we successfully diagonalized the input matrices for the eight suspended optics of the 40m prototype. This document describes the method for diagonalization and the experiment results.

# 1 Introduction

The main purpose of diagonalizing input matrices is to optimize the digital suspension control servos to minimize the net motions of the suspended optics [1]. The relatively precise diagonalization of them is also necessary to prevent servo instability by the cross-coupling between servo loops for different degrees of freedom and necessary to diagonalize the output matrices, so that the length control system does not excite pitch and yaw.

# 2 Theory

The digital suspension controls system has five OSEM sensors of upper left(UL), upper right(UR), lower right(LR), lower left(LL), and side, and 12 input sensing matrices to produce the position, pitch, and yaw signals, as shown in figure 1. [2, 3, 4, 5]. The position $V(\text{Pos})$, pitch $V(\text{Pit})$, and yaw $V(\text{Yaw})$ sensor output are given

$$V(\text{Pos}) = a_1 V_{\text{UL}} + a_2 V_{\text{UR}} + a_3 V_{\text{LR}} + a_4 V_{\text{LL}} \tag{1}$$
$$V(\text{Pit}) = b_1 V_{\text{UL}} + b_2 V_{\text{UR}} + b_3 V_{\text{LR}} + b_4 V_{\text{LL}} \tag{2}$$
$$V(\text{Yaw}) = c_1 V_{\text{UL}} + c_2 V_{\text{UR}} + c_3 V_{\text{LR}} + c_4 V_{\text{LL}} \tag{3}$$

where $\text{a}_1$, $\text{a}_2$, ..... , $\text{c}_3$, and $\text{c}_4$ are the input sensing matrices, and $\text{V}_{\text{UL}}$, $\text{V}_{\text{UR}}$, $\text{V}_{\text{LR}}$, and $\text{V}_{\text{LL}}$ are the outputs of four sensors. (See figure 1.) Here, we are not concerned with side sensor output, since it did not affect the diagonalization of input matrices directly.
The default coefficients are

| | | | | | |
|---|---|---|---|---|---|
| $a_1$ | 1 | $b_1$ | 1 | $c_1$ | 1 |
| $a_2$ | 1 | $b_2$ | 1 | $c_2$ | -1 |
| $a_3$ | 1 | $b_3$ | -1 | $c_3$ | -1 |
| $a_4$ | 1 | $b_4$ | -1 | $c_4$ | 1 |

The condition of diagonalization is that the suspension sensor output for any degrees of freedom contains only the signal of desired degree of freedom. To realize the condition for the diagonalization, we rely on the natural pendulum motion of the suspended optics for each degree of freedom. For example, the position sensor output should not be disturbed when the mass is excited in the natural pitch motion or natural yaw motion. [1] This condition is written as follows.
When the mass is excited only in the natural position motion, the pitch and yaw sensor outputs should be zero:

$$V(\text{Pit})_{\text{Pos}} = b_1 V_{\text{UL,Pos}} + b_2 V_{\text{UR,Pos}} - b_3 V_{\text{LR,Pos}} - b_4 V_{\text{LL,Pos}} = 0 \tag{4}$$
$$V(\text{Yaw})_{\text{Pos}} = c_1 V_{\text{UL,Pos}} - c_2 V_{\text{UR,Pos}} - c_3 V_{\text{LR,Pos}} + c_4 V_{\text{LL,Pos}} = 0 \tag{5}$$

When the mass is excited only in the natural pitch motion, the position and yaw sensor outputs should be zero:

[1]The natural position motion and natural pitch motion couple with each other in a system of a suspended rigid body. The resultant combined motions are no longer pure position or pitch motions. However, the two motions are still mostly the respective motions of position and pitch. Thus, we keep calling the combined motions the position and pitch motions, respectively.

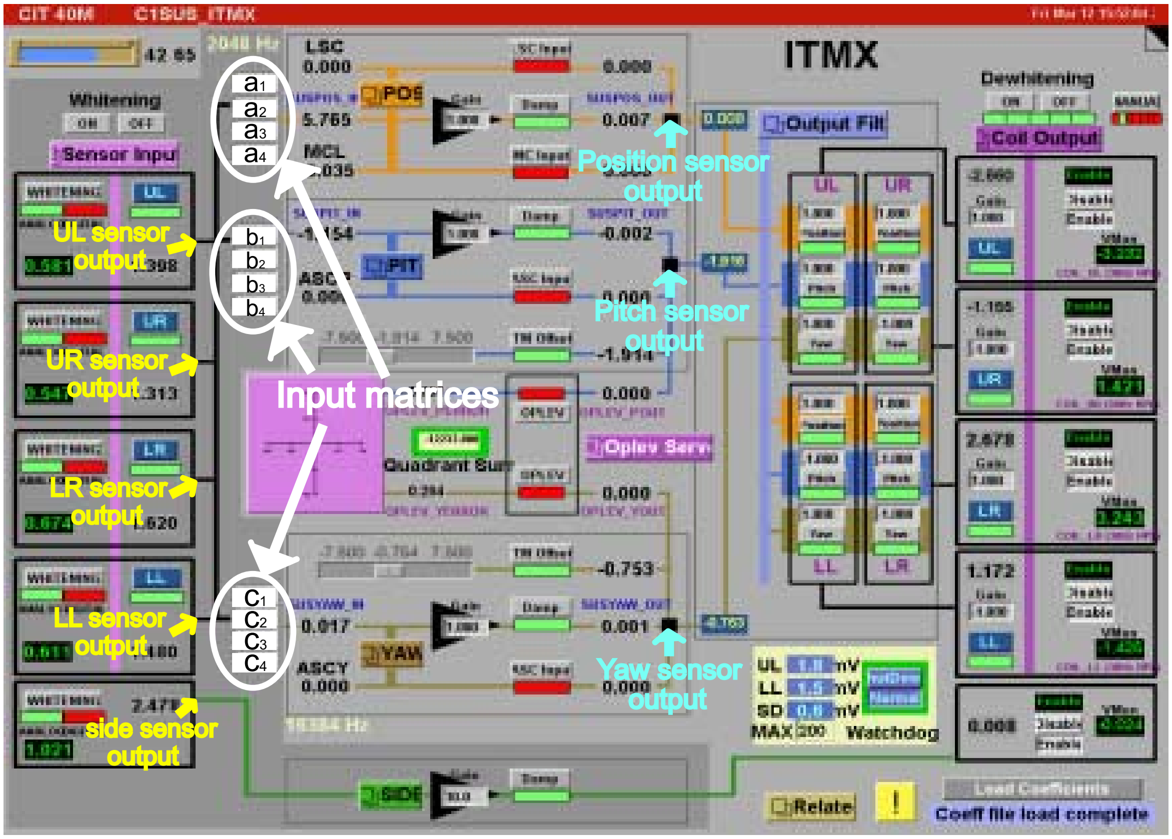


Fig. 1: Epics screen of ITMX for the digital suspension controls system.

$$
\begin{aligned}
V(\mathrm{Pos})_{\mathrm{Pit}} &= a_1 V_{\mathrm{UL,Pit}} + a_2 V_{\mathrm{UR,Pit}} + a_3 V_{\mathrm{LR,Pit}} + a_4 V_{\mathrm{LL,Pit}} = 0 && (6)\\
V(\mathrm{Yaw})_{\mathrm{Pit}} &= c_1 V_{\mathrm{UL,Pit}} - c_2 V_{\mathrm{UR,Pit}} - c_3 V_{\mathrm{LR,Pit}} + c_4 V_{\mathrm{LL,Pit}} = 0 && (7)
\end{aligned}
$$

When the mass is excited only in the natural yaw motion, the position and pitch sensor outputs should be zero:

$$
\begin{aligned}
V(\mathrm{Pos})_{\mathrm{Yaw}} &= a_1 V_{\mathrm{UL,Yaw}} + a_2 V_{\mathrm{UR,Yaw}} + a_3 V_{\mathrm{LR,Yaw}} + a_4 V_{\mathrm{LL,Yaw}} = 0 && (8)\\
V(\mathrm{Pit})_{\mathrm{Yaw}} &= b_1 V_{\mathrm{UL,Yaw}} + b_2 V_{\mathrm{UR,Yaw}} - b_3 V_{\mathrm{LR,Yaw}} - b_4 V_{\mathrm{LL,Yaw}} = 0 && (9)
\end{aligned}
$$

Twelve input matrix coefficients need to be determined by the six equations (4), (5), ....., and (9). In order to ensure a unique solution, we will add the following conditions, which will make the value of coefficients realistic.
First conditions normalize the coefficients by assigning unity to one of the position, pitch, and yaw input matrix coefficients

$$
\begin{aligned}
\mathrm{a_i} &= 1 && (10)\\
\mathrm{b_j} &= 1 && (11)\\
\mathrm{c_k} &= 1 && (12)
\end{aligned}
$$

Here, i, j, and k are either 1, 2, 3, or 4.

The second conditions make all the input matrix coefficients as close to plus or minus unity as possible

$$a_1 + a_2 + a_3 + a_4 = 4 \tag{13}$$

$$b_1 + b_2 - b_3 - b_4 = 4 \tag{14}$$

$$c_1 - c_2 - c_3 + c_4 = 4 \tag{15}$$

# 3 Determination of Input Matrix Coefficients

The OSEM sensor outputs should be measured to solve the equations, when the mass is excited in the natural motions of position, pitch, and yaw.
The way to measure the OSEM sensor outputs in the natural motion of position, pitch, and yaw was as follows.
First, in order to excite the position natural motion of the mass, we flipped the polarity of the damping servo for the position mode for a few seconds until the excited position motion of the mass reaches approximately 10% of the sensor range. Then we turned off the damping system for the position mode. Note that the damping for the other modes, pitch and yaw was always active. Secondly each of the four OSEM sensor outputs was measured in peak-to-peak value using Dataviewer. (See figure 2.) It should be noted that we took the measurement for all the four outputs at the same time, because the excited motion gradually grows or dies away due to the cross-coupling effect.
The following example is to determine the pitch input matrix coefficients for ITMY, when the position and yaw natural motions were excited separately. The input matrix coefficients are appropriated for frequencies near the pendulum resonances.

| | |
|---|---|
| $V_{Pos,UL}$= 0.0335 | $V_{Yaw,UL}$= 0.0226 |
| $V_{Pos,UR}$= 0.0261 | $V_{Yaw,UR}$= -0.0237 |
| $V_{Pos,LR}$= 0.0243 | $V_{Yaw,LR}$= -0.0213 |
| $V_{Pos,LL}$= 0.0234 | $V_{Yaw,LL}$= 0.0216 |

Substituting the above values to the equations (4) and (9), together with the equations (11) and (14), we obtained the pitch input matrix coefficients.

$$\begin{aligned} S(Pit)_{Pos} &= 0.0335 \cdot b_1 + 0.0261 \cdot b_2 + 0.0243 \cdot b_3 + 0.0234 \cdot b_4 = 0 \\ S(Pit)_{Yaw} &= 0.02257 \cdot b_1 - 0.0237 \cdot b_2 - 0.0213 \cdot b_3 + 0.0216 \cdot b_4 = 0 \\ 4 &= b_1 + b_2 - b_3 - b_4 \\ 1 &= b_2 \end{aligned}$$

$$b_1 = 0.796, \quad b_2 = 1, \quad b_3 = -1.244, \quad b_4 = -0.960$$

Here we choose $b_2$ to be unity, because $b_2$ is the closest of the average of the four b-coefficients.
The input matrix coefficients of position, pitch, and yaw for eight suspended optics using this method are summarized in Table 1 to Table 8. It can be seen from the Tables that the optimized input matrix coefficients vary form 0.6 to 1.5.
Incidentally, there are ten suspended optics at the 40m lab. Now SRM and PRM are misaligned for convenience of experiment, therefore input matrix coefficients for SRM and PRM are not yet adjusted.

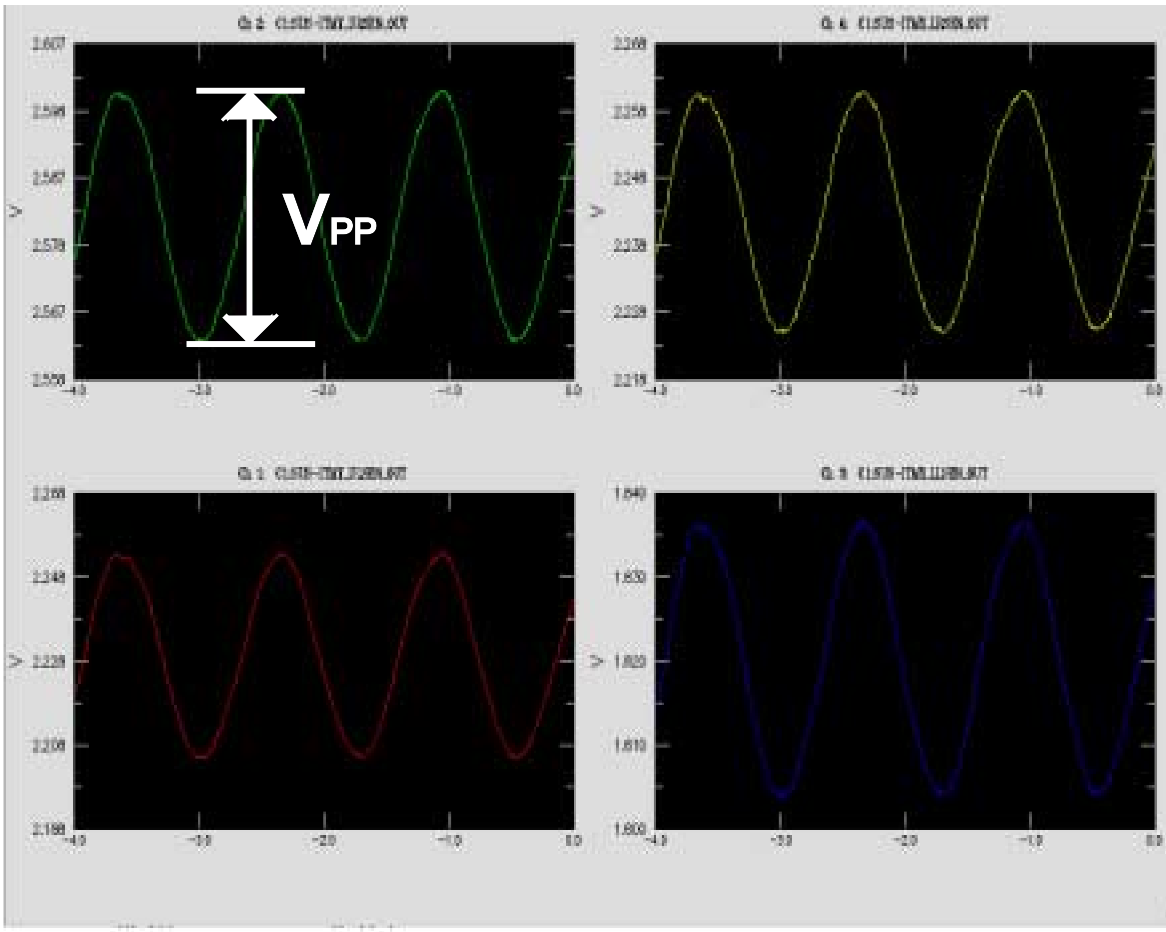


Fig. 2: The peak to peak outputs of ITMY OSEM sensors of UL, UR, LR, and LL in the dataviewer.

| ITMX POS | | ITMX PIT | | ITMX YAW | |
|---|---|---|---|---|---|
| $a_1$ | 1.000 | $b_1$ | 0.750 | $c_1$ | 0.793 |
| $a_2$ | 1.487 | $b_2$ | 1.441 | $c_2$ | -1.357 |
| $a_3$ | 0.787 | $b_3$ | -0.809 | $c_3$ | -1.000 |
| $a_4$ | 0.727 | $b_4$ | -1.000 | $c_4$ | 0.851 |

Table 1: Input matrix coefficients for ITMX.

| ITMY POS | | ITMY PIT | | ITMY YAW | |
|---|---|---|---|---|---|
| $a_1$ | 1.031 | $b_1$ | 0.796 | $c_1$ | 0.900 |
| $a_2$ | 0.983 | $b_2$ | 1.000 | $c_2$ | -1.336 |
| $a_3$ | 1.000 | $b_3$ | -1.244 | $c_3$ | -0.764 |
| $a_4$ | 0.986 | $b_4$ | -0.960 | $c_4$ | 1.000 |

Table 2: Input matrix coefficients for ITMY.

| ETMX POS | | ETMX PIT | | ETMX YAW | |
|---|---|---|---|---|---|
| $a_1$ | 0.682 | $b_1$ | 0.983 | $c_1$ | 1.364 |
| $a_2$ | 1.490 | $b_2$ | 1.000 | $c_2$ | -0.714 |
| $a_3$ | 1.000 | $b_3$ | -0.939 | $c_3$ | -0.922 |
| $a_4$ | 0.828 | $b_4$ | -1.079 | $c_4$ | 1.000 |

Table 3: Input matrix coefficients for ETMX.

| ETMY POS | | ETMY PIT | | ETMY YAW | |
|---|---|---|---|---|---|
| $a_1$ | 0.864 | $b_1$ | 1.000 | $c_1$ | 0.951 |
| $a_2$ | 1.401 | $b_2$ | 0.941 | $c_2$ | -1.000 |
| $a_3$ | 0.729 | $b_3$ | -0.853 | $c_3$ | -1.181 |
| $a_4$ | 1.000 | $b_4$ | -1.206 | $c_4$ | 0.868 |

Table 4: Input matrix coefficients for ETMY.

| BS POS | | BS PIT | | BS YAW | |
|---|---|---|---|---|---|
| $a_1$ | 0.982 | $b_1$ | 0.911 | $c_1$ | 0.861 |
| $a_2$ | 0.843 | $b_2$ | 1.000 | $c_2$ | -1.000 |
| $a_3$ | 1.176 | $b_3$ | -0.982 | $c_3$ | -0.896 |
| $a_4$ | 1.000 | $b_4$ | -1.107 | $c_4$ | 1.243 |

Table 5: Input matrix coefficients for BS.

| MC1 POS | | MC1 PIT | | MC1 YAW | |
|---|---|---|---|---|---|
| $a_1$ | 1.180 | $b_1$ | 0.889 | $c_1$ | 1.253 |
| $a_2$ | 1.000 | $b_2$ | 1.000 | $c_2$ | -1.000 |
| $a_3$ | 0.930 | $b_3$ | -0.806 | $c_3$ | -1.007 |
| $a_4$ | 0.890 | $b_4$ | -1.305 | $c_4$ | 0.740 |

Table 6: Input matrix coefficients for MC1.

| MC2 POS | | MC2 PIT | | MC2 YAW | |
|---|---|---|---|---|---|
| $a_1$ | 0.704 | $b_1$ | 0.691 | $c_1$ | 0.662 |
| $a_2$ | 1.561 | $b_2$ | 1.000 | $c_2$ | -1.350 |
| $a_3$ | 1.000 | $b_3$ | -1.225 | $c_3$ | -1.000 |
| $a_4$ | 0.735 | $b_4$ | -1.083 | $c_4$ | 0.988 |

Table 7: Input matrix coefficients for MC2.

| MC3 POS | | MC3 PIT | | MC3 YAW | |
|---|---|---|---|---|---|
| $a_1$ | 1.000 | $b_1$ | 1.002 | $c_1$ | 1.000 |
| $a_2$ | 1.360 | $b_2$ | 0.659 | $c_2$ | -0.956 |
| $a_3$ | 0.725 | $b_3$ | -1.000 | $c_3$ | -1.011 |
| $a_4$ | 0.915 | $b_4$ | -1.339 | $c_4$ | 0.943 |

Table 8: Input matrix coefficients for MC3.

# 4 Measurement of Cross-coupling

The spectra of the position, pitch, and yaw signal outputs for eight suspended optics were taken, in order to verify the efficiency of the diagonalization, with all the input matrix coefficients changed from the default value, 1 or $-1$, to the obtained values.

| Resonance peak amplitude \ Excitation | POS | PIT | YAW |
|---|---|---|---|
| POS | 100 | 0.29 | 0.17 |
| PIT | 0.14 | 100 | 0.79 |
| YAW | 0.45 | 2.3 | 100 |

Table 9: Cross-coupling for ETMY, in percentage, is determined for the ratios of resonance peak amplitude in the three degrees of freedom.

Figure 3 shows the spectra of position, pitch, and yaw sensor outputs for ETMY. The top figure is when the natural position mode was excited, the middle one is when the natural pitch mode was excited, and the bottom one is when the natural yaw mode was excited.

The resultant cross-coupling ratio is summarized in Table 9. The figures in Table 9 are expressed as percentages. The value 0.79% (third raw, fourth column) is the cross-coupling of the yaw motion into the pitch sensor output.

These spectra were taken within 10 minutes after the experiment for determining the input matrix coefficients, described in section 3, was done. Table 10 to Table 17 show the cross-coupling for eight suspended optics. We found that the largest cross-coupling was 9.7% and the smallest cross-coupling was 0.20% for eight suspensions.

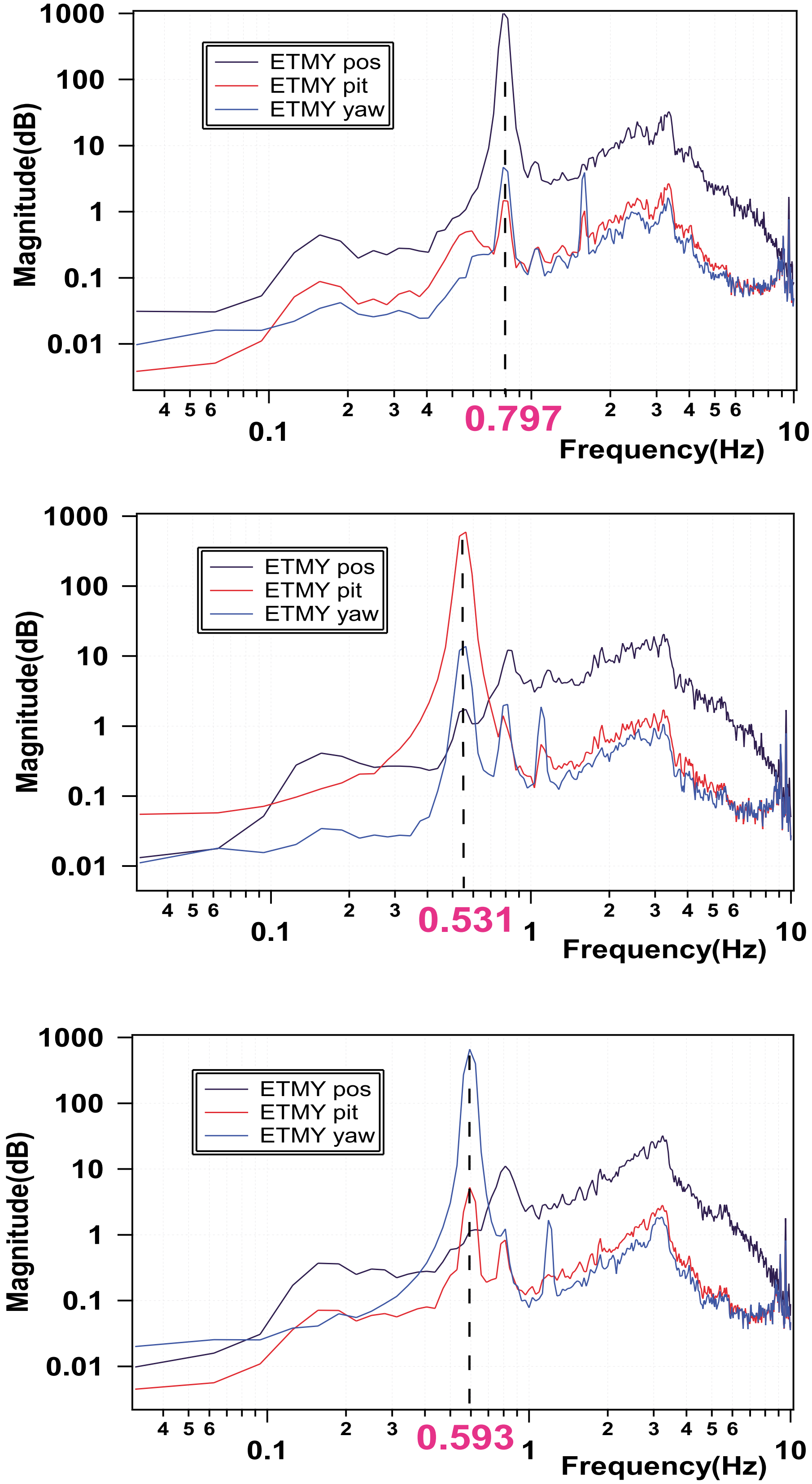


Fig. 3: Spectra for ETMY.

| Excitation / Resonance peak amplitude | POS | PIT | YAW |
|---|---|---|---|
| POS | 100 | 1.7 | 4.8 |
| PIT | 2.0 | 100 | 1.8 |
| YAW | 0.57 | 0.21 | 100 |

Table 10: Cross-coupling for ITMX.

| Excitation / Resonance peak amplitude | POS | PIT | YAW |
|---|---|---|---|
| POS | 100 | 1.9 | 1.8 |
| PIT | 0.37 | 100 | 0.73 |
| YAW | 0.20 | 0.97 | 100 |

Table 11: Cross-coupling for ITMY.

| Excitation / Resonance peak amplitude | POS | PIT | YAW |
|---|---|---|---|
| POS | 100 | 1.7 | 9.0 |
| PIT | 2.6 | 100 | 1.3 |
| YAW | 3.0 | 9.7 | 100 |

Table 12: Cross-coupling for ETMX.

| Excitation / Resonance peak amplitude | POS | PIT | YAW |
|---|---|---|---|
| POS | 100 | 0.29 | 0.17 |
| PIT | 0.14 | 100 | 0.79 |
| YAW | 0.45 | 2.3 | 100 |

Table 13: Cross-coupling for ETMY.

| Excitation / Resonance peak amplitude | POS | PIT | YAW |
|---|---|---|---|
| POS | 100 | 3.7 | 1.3 |
| PIT | 1.8 | 100 | 8.9 |
| YAW | 4.1 | 4.3 | 100 |

Table 14: Cross-coupling for BS.

| Excitation / Resonance peak amplitude | POS | PIT | YAW |
|---|---|---|---|
| POS | 100 | 3.8 | 1.2 |
| PIT | 4.1 | 100 | 8.7 |
| YAW | 1.1 | 5.1 | 100 |

Table 15: Cross-coupling for MC1.

| Excitation / Resonance peak amplitude | POS | PIT | YAW |
|---|---|---|---|
| POS | 100 | 3.8 | 5.2 |
| PIT | 8.1 | 100 | 3.4 |
| YAW | 8.3 | 1.9 | 100 |

Table 16: Cross-coupling for MC2.

| Excitation / Resonance peak amplitude | POS | PIT | YAW |
|---|---|---|---|
| POS | 100 | 1.6 | 2.4 |
| PIT | 3.7 | 100 | 0.91 |
| YAW | 0.90 | 4.1 | 100 |

Table 17: Cross-coupling for MC3.

# 5 Resonant Frequencies

We also ascertained the resonant frequencies of the position, pitch, and yaw mode of eight suspended optics. Table 18 and Table 19 were made to compare the measured and designed resonant frequencies [6]. Note that the bandwidth from the experiment was 0.0117.

| | ITMX | ITMY | ETMX | ETMY | BS | MC1 | MC2 | MC3 |
|---|---|---|---|---|---|---|---|---|
| Position | .800 | .800 | .800 | .800 | 1.000 | 1.000 | 1.000 | 1.000 |
| Pit | .500 | .500 | .500 | .500 | .744 | .744 | .744 | .744 |
| Yaw | .600 | .600 | .600 | .600 | .856 | .856 | .856 | .856 |

Table 18: Designed resonant frequencies of position, pitch, and yaw for the 40m prototype [6].

| | ITMX | ITMY | ETMX | ETMY | BS | MC1 | MC2 | MC3 |
|---|---|---|---|---|---|---|---|---|
| Position | .797 | .789 | .797 | .793 | .969 | .977 | .977 | .980 |
| Pit | .531 | .531 | .586 | .531 | .750 | .660 | .703 | .660 |
| Yaw | .594 | .594 | .605 | .602 | .813 | .836 | .816 | .816 |

Table 19: Measured resonant frequencies of position, pitch, and yaw.

# 6 Stability

We took all the spectra for cross-coupling to check the stability of the input matrix diagonalization two weeks later. We found that only the cross-coupling ratio for ITMX changed very much. Therefore, we readjusted input matrix coefficients for ITMX. Table 20 is input matrix coefficients for ITMX after readjusting.

| ITMY POS | | ITMY PIT | | ITMY YAW | |
|---|---|---|---|---|---|
| $a_1$ | 1.031 | $b_1$ | 0.796 | $c_1$ | 0.900 |
| $a_2$ | 0.983 | $b_2$ | 1.000 | $c_2$ | -1.336 |
| $a_3$ | 1.000 | $b_3$ | -1.244 | $c_3$ | -0.764 |
| $a_4$ | 0.986 | $b_4$ | -0.960 | $c_4$ | 1.000 |

Table 20: Input matrix coefficients for ITMY.

Table 21 to Table 28 represent the results. According to Table 10 to Table 17 and Table 21 to Table 28, it was found that the cross-coupling of the input matrices is mostly degraded somewhat, but all of them are still less than 10%.

| Resonance peak amplitude \ Excitation | POS | PIT | YAW |
|---|---|---|---|
| POS | 100 | 3.7 | 3.6 |
| PIT | 10 | 100 | 0.82 |
| YAW | 7.8 | 0.95 | 100 |

Table 21: Cross-coupling for ITMX.

| Resonance peak amplitude \ Excitation | POS | PIT | YAW |
|---|---|---|---|
| POS | 100 | 2.3 | 3.5 |
| PIT | 0.25 | 100 | 1.9 |
| YAW | 2.2 | 1.8 | 100 |

Table 22: Cross-coupling for ITMY.

| Resonance peak amplitude \ Excitation | POS | PIT | YAW |
|---|---|---|---|
| POS | 100 | 0.78 | 6.8 |
| PIT | 4.1 | 100 | 6.3 |
| YAW | 1.1 | 5.1 | 100 |

Table 23: Cross-coupling for ETMX.

| Resonance peak amplitude \ Excitation | POS | PIT | YAW |
|---|---|---|---|
| POS | 100 | 1.9 | 1.8 |
| PIT | 0.90 | 100 | 2.8 |
| YAW | 2.2 | 7.5 | 100 |

Table 24: Cross-coupling for ETMY.

| Resonance peak amplitude \ Excitation | POS | PIT | YAW |
|---|---|---|---|
| POS | 100 | 3.8 | 1.5 |
| PIT | 1.7 | 100 | 8.9 |
| YAW | 4.3 | 4.3 | 100 |

Table 25: Cross-coupling for BS.

| Resonance peak amplitude \ Excitation | POS | PIT | YAW |
|---|---|---|---|
| POS | 100 | 2.5 | 2.0 |
| PIT | 3.3 | 100 | 8.7 |
| YAW | 0.53 | 5.1 | 100 |

Table 26: Cross-coupling for MC1.

| Resonance peak amplitude \ Excitation | POS | PIT | YAW |
|---|---|---|---|
| POS | 100 | 3.9 | 5.2 |
| PIT | 7.7 | 100 | 3.3 |
| YAW | 8.5 | 1.9 | 100 |

Table 27: Cross-coupling for MC2.

| Resonance peak amplitude \ Excitation | POS | PIT | YAW |
|---|---|---|---|
| POS | 100 | 1.5 | 2.7 |
| PIT | 4.3 | 100 | 0.79 |
| YAW | 1.1 | 4.4 | 100 |

Table 28: Cross-coupling for MC3.

# 7 Conclusions

The newly developed method proved to be helpful for diagonalization of the input matrix of the suspension system. It typically suppressed the cross-coupling of the input matrix below a few percentages. It was concluded that adjusting input matrices within a cross-coupling of 2% is meaningless, since cross-coupling of input matrices was degraded by up to 2% in two weeks.
The resonant frequencies of the position, pitch, and yaw mode of ITM, ETMY, BS, and MC were measured to be within 12% from the designed parameters.

# 8 Future work

- Input matrix for SRM and PRM
- Optimize damping gains
- Output matrix diagonalization
- Continued monitoring of performance of damping (RMS motion of all 10 optics in all 3+side degree of freedom)

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