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Frequency Analysis of the Quadruple Pendulum Structure

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Abstract

A natural mode and frequency analysis is performed on a proposed structure to support which cages or restrains and supports the quadruple pendulum ofr advanced LIGO. The (as yet unvetted) requirement is that the first natural frequency should be greater than 150 Hz, so that interaction of this structure with the SEI system does not destabilize the SEI control[⊕]. The primary purpose of this analysis of an early structural concept is to provide a mass estimate for the purpose of establishing optical table mass budgets. The issue of sufficient stiffness in the attachment provisions to the optics table is also addressed.

Revision 00: 9 Mar 2003, Limited, draft release.

Revision 01: 1 Apr 2003, Limited draft release. Revised with the addition of (considerable) nonstructural mass added to the structure. The design concept still does not meet the minimum frequency requirement but is closer. Consideration for the bolted interface is included. A better approximation of the boundary condition at the optics table interface is still needed.

Revision 02: 2 Apr 2003, minor word smithing.

Revision 03: 19 Feb 2004: (a) Correction to the plate bending formula results (had used material density instead of plate areal density; coefficients for the SF boundary condition case were incorrect), (b) Improved frequency analysis of the stiffened plate design, (c) addition of an appendix discussing the limitations of solid elements for analysis of plate/shell structures

1 Initial Design Concept

A design concept for the quad structure based on a space truss with adequate openings for the optical beam and possibly for assembly (this needs to be reviewed) is shown in Figure 1 (by Larry Jones). Some elements of the truss would clearly need to be removable for initial assembly.

2 Modes of the Initial Design Concept

The first three modes and frequencies for the initial concept are shown below (figures 2 through 4), where the boundary condition at the base (actually top of the structure where it attaches to the optics table) is optimistically assumed to be clamped. The finite element analysis (I-DEAS, version 9) is a beam and plate approximation so that parameters can be quickly changed and the resulting effects quickly evaluated. It is apparent that the first resonance (first lateral bending mode) is far less than the required value (58 Hz vs 150 Hz).

The finite element result is in close agreement with a first bending mode of a beam with cross-sectional properties of the four tubes which form the corners of the lower (upper in the mode diagrams) section, i.e. the 3 inch square tubes with 0.125 inch thick walls:

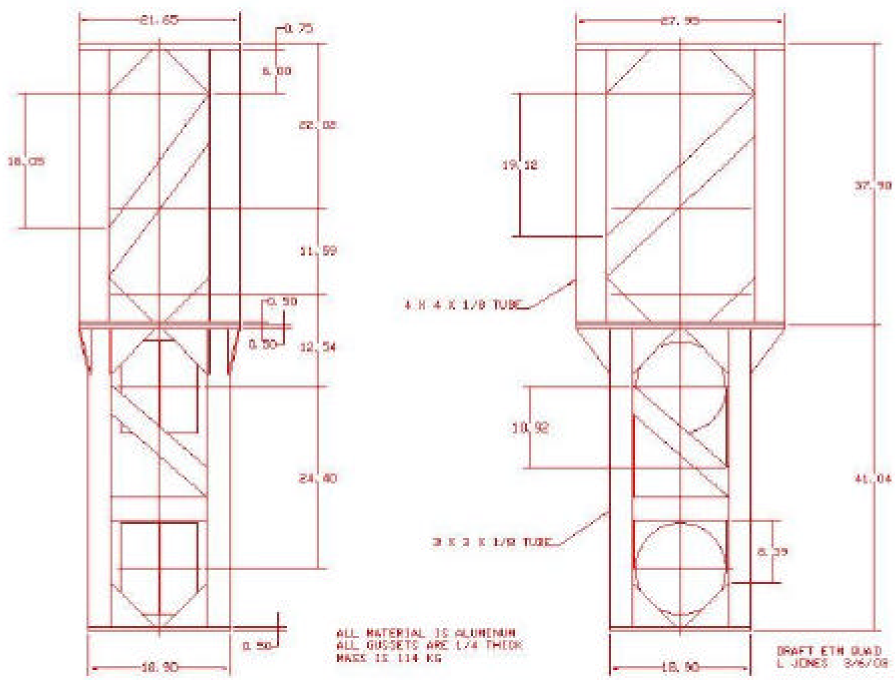
$$f = \frac{m^2}{2p}\sqrt{\frac{EI}{ml^4}} = 60Hz$$

where f is the first resonant frequency of the beam, $\mu = 1.875$ is the eigenvalue for a cantilevered beam, E = 69 GPa = 69 x 10^6 mN/mm^2 is the elastic modulus for aluminum, I = 1.55 x 10^8 mm^4 is

[⊕] Alternatively a very light structure might be acceptable if the interaction with the SEI platform was small due to the large impedance mis-match. For now the requirement to have no payload structural resonances close to the SEI upper unit gain point is taken as the requirement, i.e. > 150 Hz.

the area moment of inertia of the cross-section formed by the 4 tubes in the lower section of the design, l = 2005 mm is the beam overall length[1] and m = 0.0569 kg/mm is the lineal density of the beam. This favorable agreement means that the cross-bracing in the design is effective at tying the four individual legs together, effectively maintaining strain continuity in the structure.

Figure 1: Initial design concept for the quadruple pendulum structure (dimensions are in inches)



[1] Although the overall quadruple pendulum structure length was initially estimated to be 2.1 m, a recent decision was made to set the length to 2.005 m per N. Robertson, "Investigation of Wire Lengths in Advanced LIGO Quadruple Pendulum Design for ETM/ITM", LIGO-Tpending, 1/26/2004.

Figure 2: First mode of the initial design concept (lateral bending, 58 Hz)

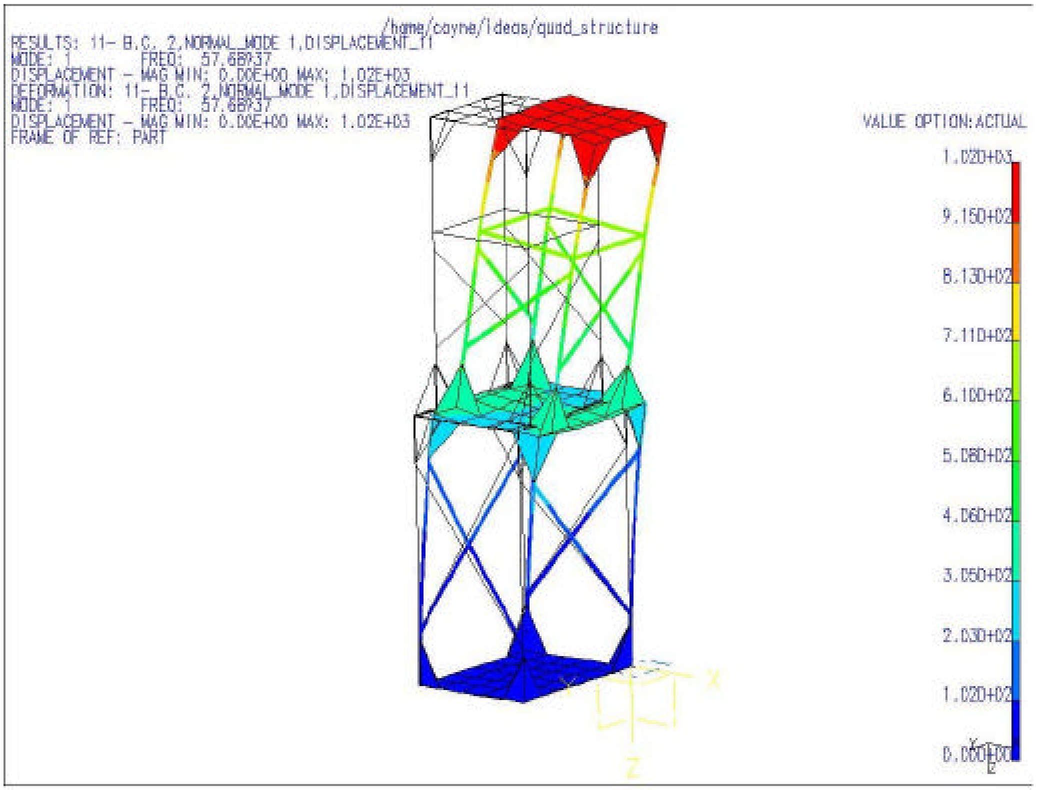


Figure 3: Second mode of the initial design concept (transverse bending, 73 Hz)

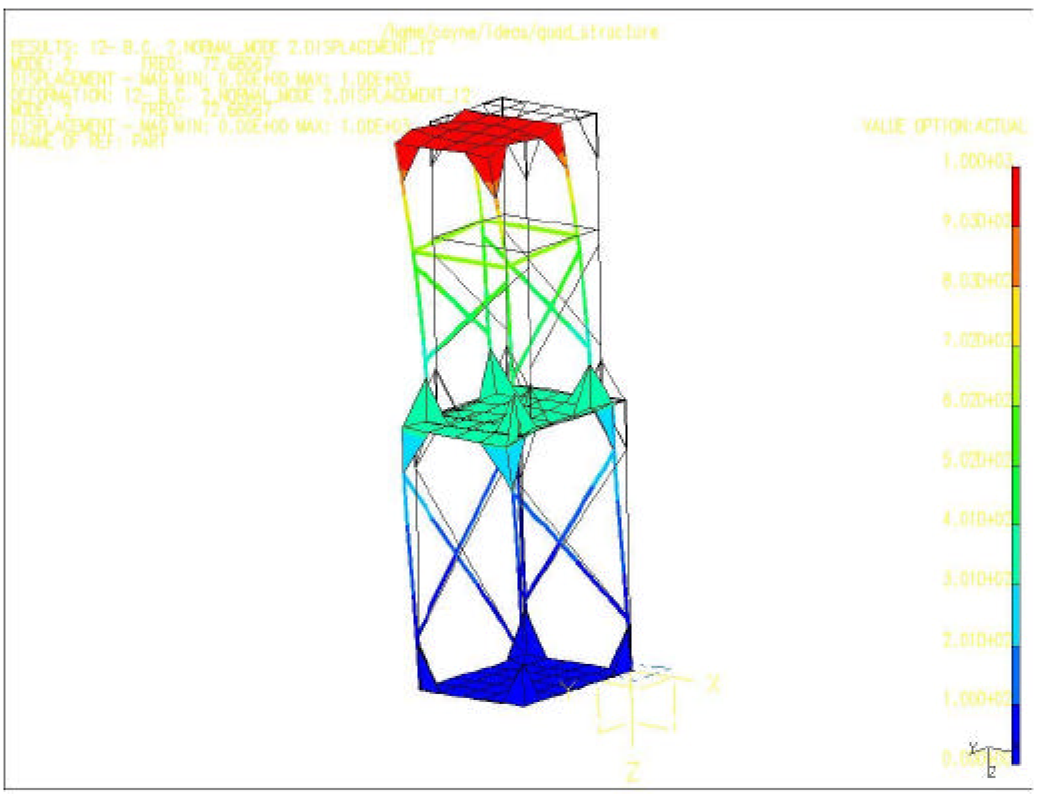
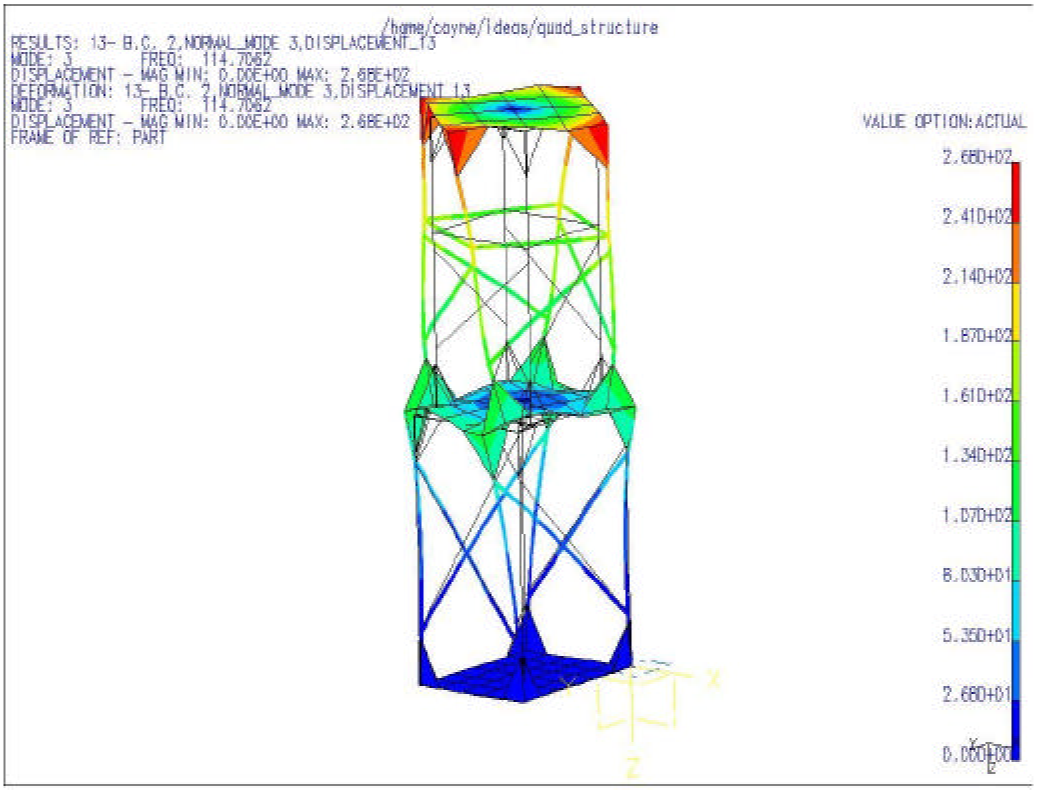


Figure 4: Third mode of the initial design concept (torsion, 115 Hz)



Using the above beam equation, it appears that Beryllium or Metal Matrix Composites would be required to achieve the required first frequency based on the original design concept (Table 1). Since these materials are expensive, design alternatives are sought next.

Table 1: Estimated 1st Natural Frequency for the initial design concept with various materials (clamped attachment)

Material	**Elastic Modulus (GPa)**	**density g/cc**	**Specific Modulus (GPa/g/cc)**	**Estimated 1st Frequency (Hz) (for the initial quad structure concept)**
Stainless Steel 303	193	8	24	58
Aluminum 6061-T6	69	2.7	26	60
Titanium alloy	120	4.56	26	61
Aluminum/Alumina Al/Al2O3 MMC (MMCC)	130	3.1	42	76
Aluminum/Silicon Carbide MMC (PPC)	220	3.01	73	101
Beryllium/BeO Metal Matrix Composite (Brush Wellman E60)	330	2.52	131	135
Beryllium S-200 tubing	303	1.84	165	151

3 Parameter Variations on the Initial Design Concept

A number of perturbations on the basic initial design concept were explored in attempts to increase the first natural resonance. The results are shown in Table 1. None of these modest perturbations in the design are effective at significantly increasing the first resonance frequency. The reason is apparent if the beam equation is used to estimate the required effective moment properties of the section:

$$\frac{EI}{m} = \frac{4p^2 f^2 l^4}{m^4} = 2.89x10^{11}\, mm^4 / s^2$$

where f = 150 Hz (the requirement) in the above equation. Since to a good approximation:

$$I = 4Ad^2$$

$$m = 4c(1+\sqrt{2})Ar$$

where A is the cross-sectional area of each of the four vertical tubes, d is the distance from the center of the tube to the center of the structure in the weak bending axis, ρ = 2.7 g/cc is the material density for aluminum, the √2 is to approximately account for the cross-bracing mass and the coefficient c = 2.35 accounts for the mass of the gussets and plates. Substituting the above expressions for I and m into the requirement for EI/m results in the following requirement for the separation of the tube centers:

$$2d = 1.67x10^6 \left(mm^2/s\right)\sqrt{\frac{r}{E}} = 662mm \;\; for\ alum$$

Note that the cross-sectional area drops out of the equation (i.e. adding more area to increase the I is compensated by increased m). This above value for 2d is to be compared to the initial design value of 404 mm [from Figure 1, 25.4 mm/in x (18.9-3)] or 448 mm [from Figure 1, 25.4 mm/in x (21.65-4)]. This is about a 50% increase in the overall dimensions and is likely not permissible due to physical constraints in the layout.

Table 2: FEA Calculated 1st Natural Frequency for the initial design concept with various minor modifications

Model #	*Description*	*Mass (kg)*	*1st Frequency (Hz)*
1	Initial design concept	101	58
2	Lower half tubes from 4 in. sq. to 3 in. sq	95	51
3	Lower half tubes from 4 in. sq. to 3 in. sq	82	47
	Upper half tubes from 3 in. sq. x 0.125 in to 2 in. sq x 0.0625 in		
4	Same as model 3, plus:	73	48
	0.125 in thick gusset plates (instead of 0.25 in thick)		
	0.250 in thick bottom plate (instead of 0.50 in thick)		
5	Same as model 4, plus:	74	50

	Gussets (0.125 in thick) added above and below horizontal bracing in the lower section above the test mass optic position		
6	Same as model 1 (original design concept), but with cross-bracing added to the test mass section. This is not practical, but serves to determine if the legs in the last section are acting together or separately	107	56
7	Same as model 1 (original design concept), except:	167	70
	Upper section tubes are 5 in. sq. x 0.25 in		
	Lower section tubes are 4 in. sq. x 0.25 in		

4 Alternative Design

Using the beam equation again for guidance, if we were to use a rectangular, thin-walled tube with outer dimensions (710 mm x 550 mm) equal to the maximum indicated in Figure 1, we find that the first frequency is 162 Hz and almost independent of the thickness. Of course to this basic structure we must add a bottom plate and a top plate. The mass of the bottom plate must be no greater than (hopefully considerably less) than the mass of the lightening holes that must be added to the structure to gain access to the interior. In addition the unsupported spans of the plates that comprise this single large rectangular tube have plate-bending frequencies, which are too low. These plates can be stiffened by ribs, but again the mass for the ribs must come from lightening holes if we are to keep the overall bending modes high. Finally the stepped approximation to a tapered beam in the original design concept still seems a valid approach (and is consistent with providing a wide structure at the top, where the blade flexures are considerably wider than the optic, and a narrower structure at the bottom section, which is close to the optic to support position stops).

The design approach for this alternate design is to place the thin shell at the outer most allowed position (relative to the central vertical, or 'structural beam' axis) and to minimize the wall thickness to the point that the shell vibrations are just over the minimum required frequency. The first vibration frequency of a simple isotropic rectangular plate is[2,3]:

$$f = \frac{2p\, k_{11}}{L_y^2}\sqrt{\frac{D}{r}}$$

where f is the frequency in Hz, the plate dimensions are L by Ly with a thickness of h, ρ is the ~~material~~ plate areal density, k_{11} is the coefficient of the first mode and $D = Eh^3/12(1-n^2)$ is the bending stiffness of the plate. The value of the coefficient, k_{11}, depends upon the boundary conditions of the plate, as indicated in the following Table.

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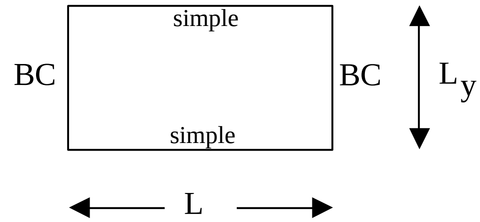
[2] W. Pilkey, P. Chang, Modern Formulas for Statics and Dynamics, McGraw Hill, 1978, pg.338.

[3] A. Leissa, Vibration of Plates, NASA SP-160.

Table 3: Coefficient k_{11}

Aspect Ratio:

$\alpha = L_y/L$



Boundary Condition (BC)	Coefficient, k_{11}
Free, Free (FF)	9.87 (ν=0.3)
Simple, Free (SF)	~~α = 1 1.5 2 2.2~~ ~~k_{11} = 12.9 17.2 23.2 26.2~~ α = 1 1.6 2 2.5 3 5 k_{11} = 11.843 14.409 16.481 19.244 22.205 35.133
Simple, Simple (SS)	$p^2(1+a^2)$
Clamped, Simple (CS)	$p^2\sqrt{1+2.33a^2+2.44a^4}$
Clamped, Clamped (CC)	$p^2\sqrt{1+2.5a^2+5.14a^4}$

The next two figures show the maximum dimension Ly as a function of plate thickness and boundary condition, for aspect ratios of 1 and 2. ~~As an example,~~

Figure 5: Rectangular Plate Maximum Dimension, L_y, vs Thickness, h, for Aspect Ratio a = 1 (Boundary conditions for lower to upper curve: FF, FS, SS, CS, CC)

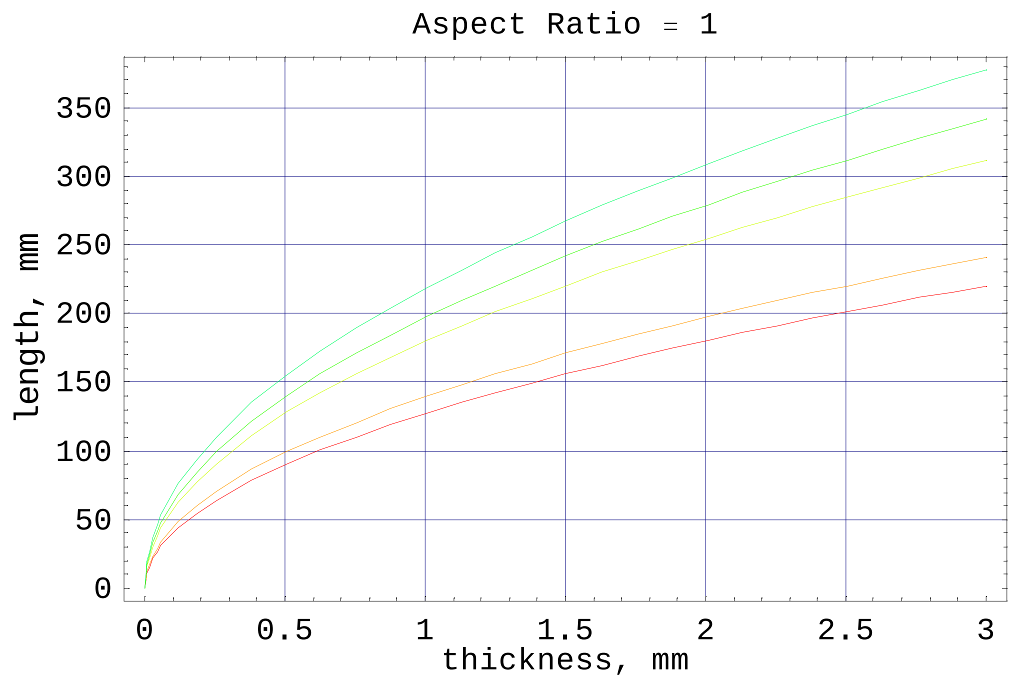
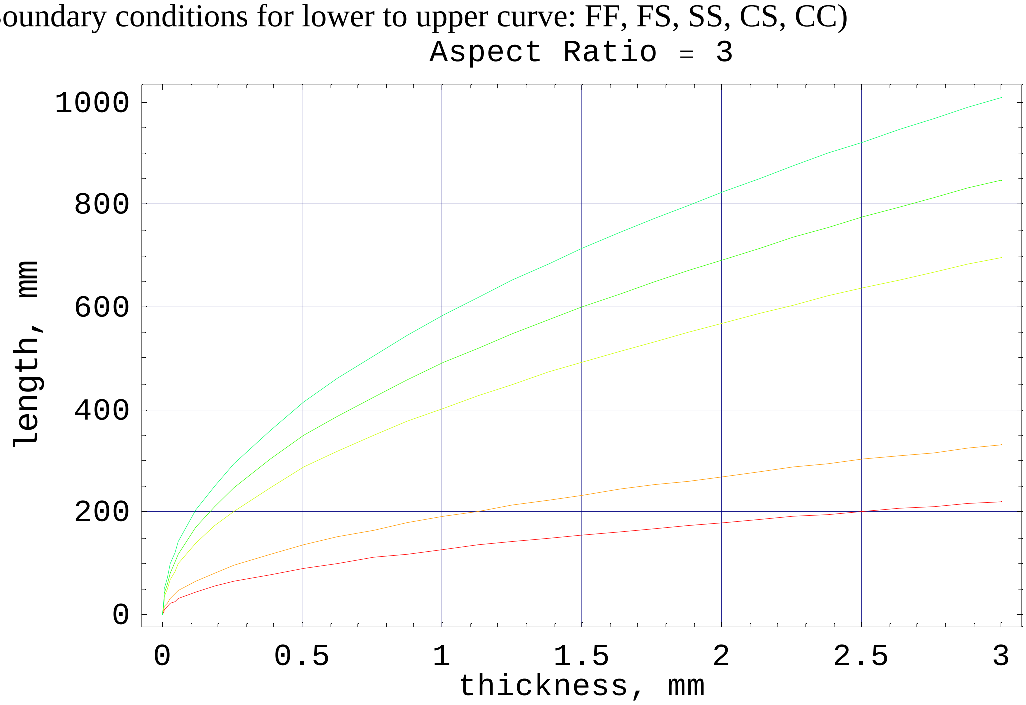


Figure 6: Rectangular Plate Maximum Dimension, L_y, vs Thickness, h, for Aspect Ratio α = 3 (Boundary conditions for lower to upper curve: FF, FS, SS, CS, CC)



As an example, for an aspect ratio of 3 and a 1 mm thick plate, the maximum dimensions for a SS-SF plate with a first frequency of 150 Hz are ~~300~~ 191 mm x ~~100~~ 64 mm. These simple plate natural frequency calculations are used to guide the design of the free spans of panels in the alternate design. The design then checked with a finite element analysis.

The basic thin shell design concept is indicated in Figure 7. The unsupported panel spans are reduced with lightening holes. In addition, ribbing has been added to reduce the plate (or thin beam) spans. The four corner posts of the upper section have been stiffened considerably to meet the minimum 150 Hz frequency requirement. Clearly for access to the suspended components during installation some sections of the "x" shaped panels will need to be separable and fastened to to the main structure.

The results of some finite element analyses (Algor) are summarized in Table 4. With 1 mm thick plate (0.039 in), with the stiffening/ribs and lightening/access holes indicated in Figure 7, the first bending resonances are 174 Hz and 182 Hz. The next natural mode is torsion of the entire structure about the vertical axis at 256 Hz. The next 2 modes are the second bending modes of the entire structure acting as a beam at 344 and 386 Hz. All local shell modes are at higher frequencies. The first 3 mode shapes are shown in the figures below. The total mass of this structure is 45 kg.

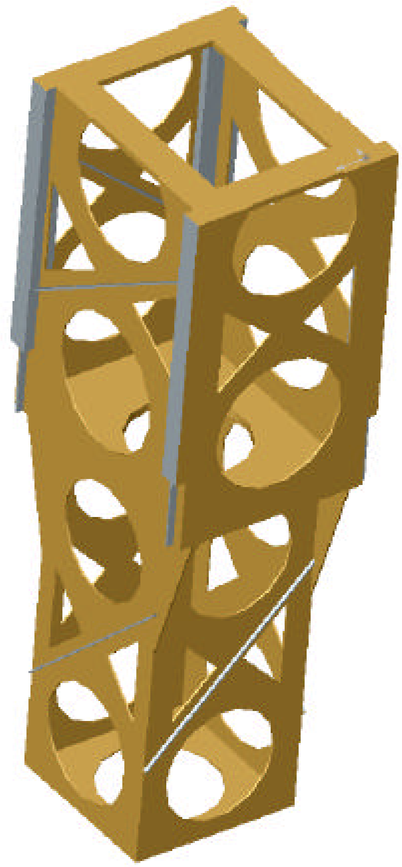
N.B: These results are with an Algor mesh employing ~17,000, 2^{nd} order tetraheral solid elements without mid-side nodes; The actual frequencies are likely to be considerably lower. Further analysis with a more representative model is reported below.

Table 4: Shell Structure Modal Analysis Results

(note that ~30k elements are required to get a converged result)

Model #	*Description*	*Mass (kg)*	*Natural Frequencies (Hz)*
1	6 mm thick plates	104	
	? elements, ? nodes		151, 173, 220, …
2	3 mm thick plates:	62	
	? elements, ? nodes		196, 221, 438, …
	? elements, ? nodes		171, 195, 324, …
	26156 elements, 12070 nodes		151, 176, 261, …
	32893 elements, 23190 nodes		143, 168, 240, 298, 339, …
3	1.5 mm thick shells & bottom plate, 3 mm thick bulkhead, 6 mm thick top plate:	50	160, 176, 246, 321, 362, …
	36925 elements, 24758 nodes		
4	1 mm thick shells & bottom plate, 1.5 mm thick bulkhead, 6 mm thick top plate:	45	174, 182, 256, 344, 386, …
	44458 elements, 26782 nodes		

Figure 7: Alternate Design



5 Corrections and Refinements

Although the basic alternative design approach seems promising, there are two corrections that are required before establishing the shell thickness and overall mass estimate:

the mass of the nonstructural additions must be included, such as (a) the coil holder, coils, magnetic dampers, etc. at the upper suspension mass, (b) the clamps at the upper mounting plate, (c) suspended mass position stops and (d) fastening hardware.

2)1) the length of the suspension structure should be 2.10 m, instead of the 2.00 m shown in Figure 1.

The total mass for the nonstructural additions to the non-suspended mass items has been very roughly estimated[4] to be 70 kg. This estimate is based upon actual weights for the elements from the small triple suspension and scaled from thereSome of this additional mass may actually be used

[4] Larry Jones and Calum Torrie, et. al., Mass Estimate of an ETM Suspension Layout, T030137-04, 28 Mar22 Jul 2003.

for structural stiffening, though much of it will not help stiffen the structure. The distribution of this mass is defined in T030137-04.~~:~~

- ~~? Clamps: the large S/S clamps used currently weigh 0.32 kg ea, with screw. Estimating that we use 10 clamps per LOS currently. Changing to aluminum and scaling to 30 clamps for the quad perimeter and adding a few for central hole clamping gives approx. 4 kg.~~
- ~~? OSEM coils: 12 current S/S assemblies weigh 5kg; we can change these to aluminum and come up with approx. 1 kg for each set of 6. I decided to ignore the eddy current dampers, as there's no design and no certainty of need.~~
- ~~? Tablecloths and mounting blocks: I estimate the mass of the current S/S assembly at 2.8 kg. We can change this to aluminum (x .35) and scale it up to the quad size (approx. x 8), giving 8 kg per position.~~
- ~~? EQ stops: the aluminum parts for the small triple was weighed out at 1 kg per position; scaling this by x 8 for the quad gives 8 kg per position.~~

~~The CG of this conglomerate would be (using spacing from Norna Robertson of a shorter quad, plus distributing the added length nearly equally):~~

~~CG = (4*1.5 + 18*60 + 16*89.5 + 16*121.4 + 16*185.0)/70 = 106 cm below the table surface.~~

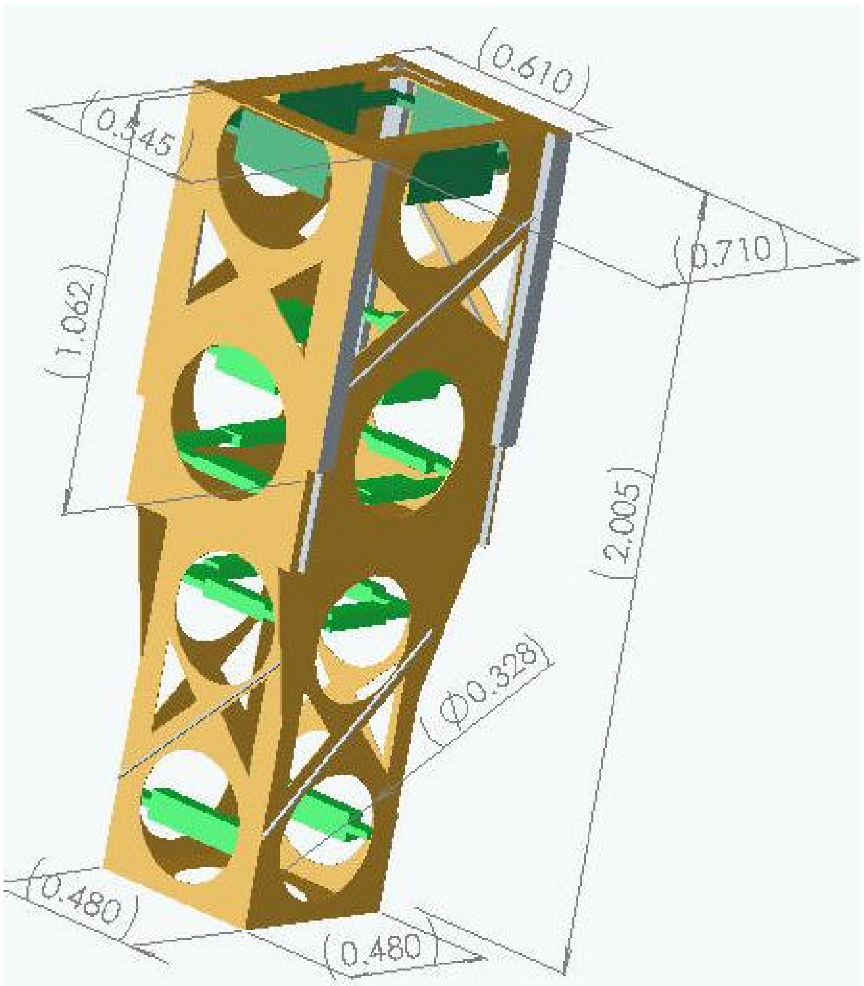
~~I suppose that it would be possible to reduce the masses below the 70 kg total with some design time, but I'm not sure that we wouldn't have to add some stiffening features to the basic frame to support the EQ stop bars. I would stay with the 70 kg to be careful. In addition, the scaling factor of 8 (2 times in each dimension) is likely an overestimate. More design work is needed to firm up the estimate of this nonstructural mass and to determine what fraction can in fact be incorporated into the structure to help with stiffness.~~

The nonstructural mass additions were made to the solid model as aluminum blocks tied into the corners of the shell structure with beams and corner blocks. The localized beam bending frequency of the "mass blocks" is about 200 Hz, i.e. above the resonance requirement of 150 Hz, so that these added masses participate in the lower frequency whole-structure modes. Clearly in an optimal structure some of the 70 kg "nonstructural" mass should be used to stiffen and lighten the overall structure. However this aspect is beyond the scope of this document.

With these changes made to the finite element model, the first whole-structure bending mode frequencies are ~~117~~ 102 Hz and ~~123~~ 124 Hz. In addition, there are a pair of modes at 117 Hz and 120 Hz associated with out-of-plane bending of the stiffened side panels of the upper section. The eigenvalue analysis was performed with Algor using solid elements with ~~1.5~~ 5 mm thick walls in the upper section and 3 mm thick walls in the lower section. The mesh consisted of ~~17,745~~ 55,535 elements ~~and 7,003 nodes~~ and is likely to have been near a converged result (though this was not verified with additional meshs; see the appendix). A shell/plate mesh woul~~d~~d be a better approximation for the thin panels, but more difficult to mesh properly (with Algor). (A shell mesh with beams or solid element stiffeners is being pursued with I-DEAS.)

The solid model is depicted in Figure 8. The first ~~two~~ four modes are depicted in Figures 9 ~~and 10~~through 12. The structure mass is ~~51~~68 Kg. It is estimated that a ~~60~~ 66 Kg structure could meet the 150 Hz requirement, assuming that ~~none~~ some of the 70 kg nonstructural mass can be used for stiffening. Further analysis would be required to verify this estimate.

Figure 8: Revised Shell Structure Concept (the green parts represent the nonstructural masses)



Mass = 132.68 kilograms

Center of mass: (meters)

Z = -0.65

Moments of inertia: (kilograms * square meters)

Taken at the center of mass and aligned with the output coordinate system.

Lxx = 41.96	Lxy = 0.01	Lxz = -0.03
Lyx = 0.01	Lyy = 44.98	Lyz = -0.02
Lzx = -0.03	Lzy = -0.02	Lzz = 14.85

Figure 9: Finite Element Mesh (solid, tetrahedra, bricks, mid-side nodes, ~50k elements)

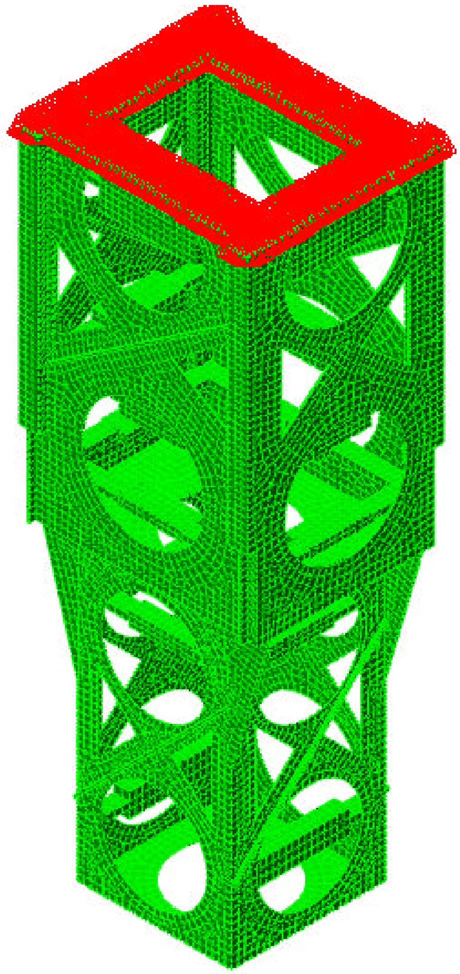


Figure ~~8~~10: 1st Mode, ~~117~~ 102 Hz (longituidinal bending)

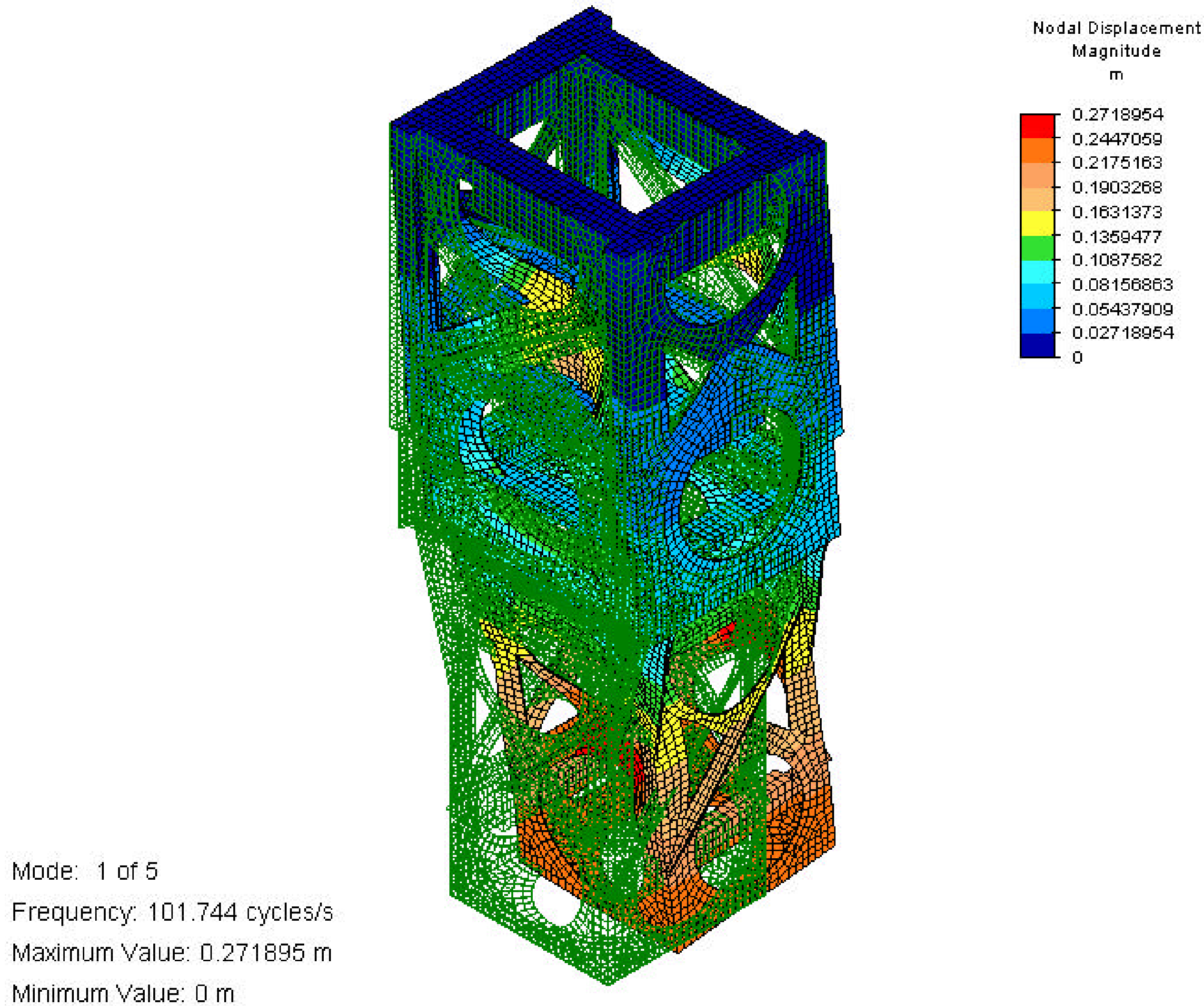


Figure ~~9~~11: 2nd mode, ~~123~~ 117 Hz (upper stiffened plate bending)

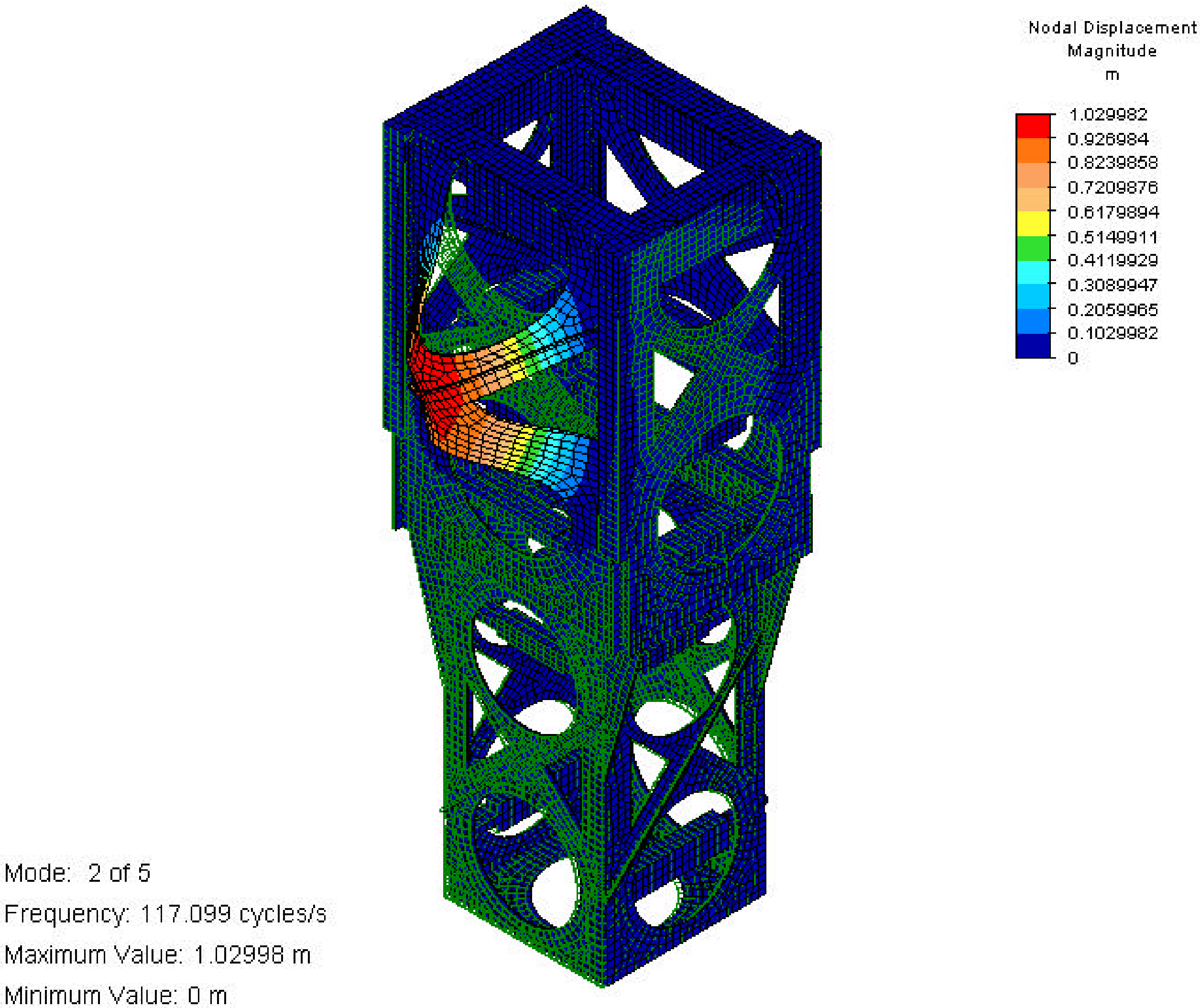


Figure 12: 3rd mode, 120 Hz (upper stiffened plate bending)

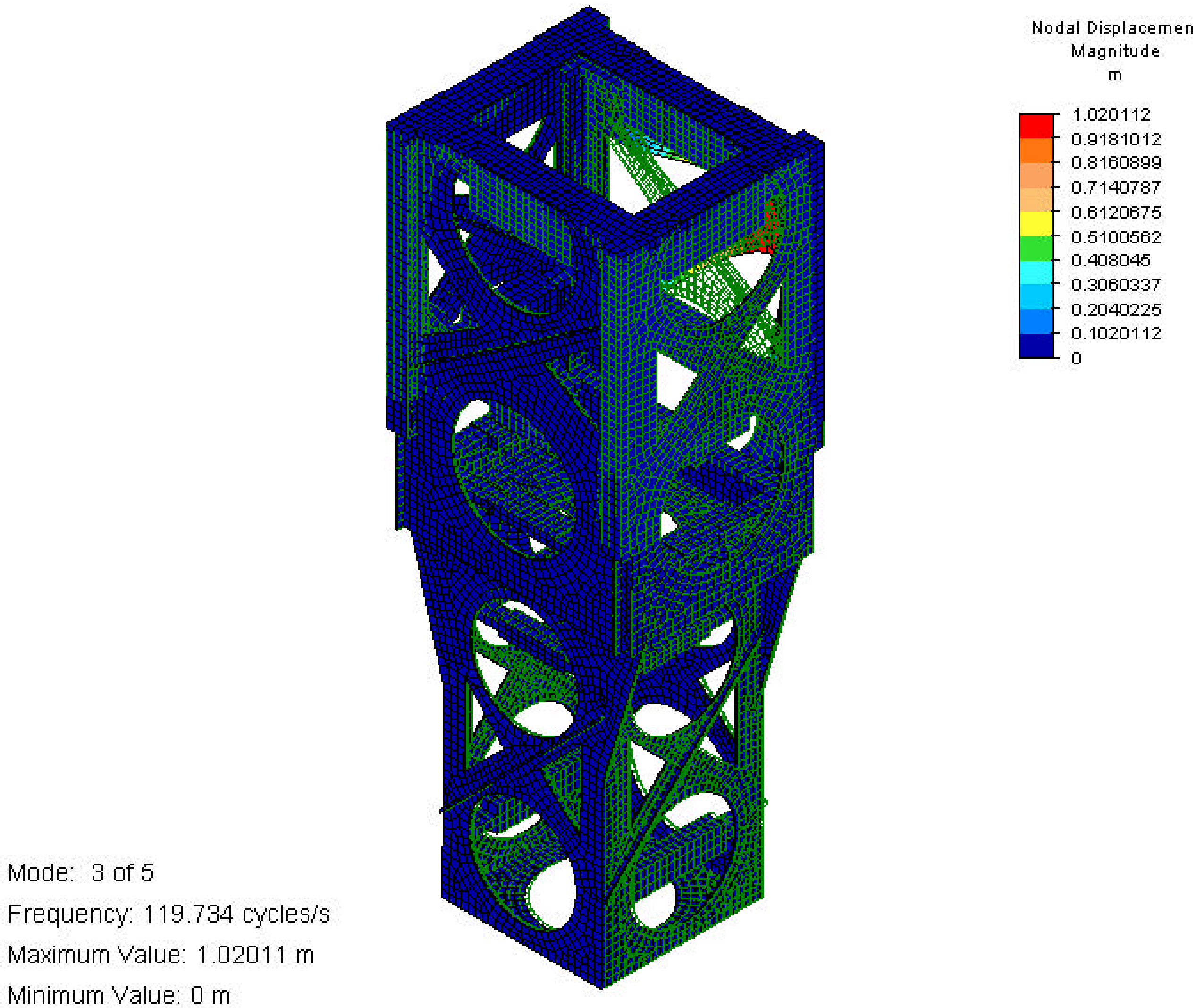


Figure 13: 4th mode, 124 Hz (transverse bending)

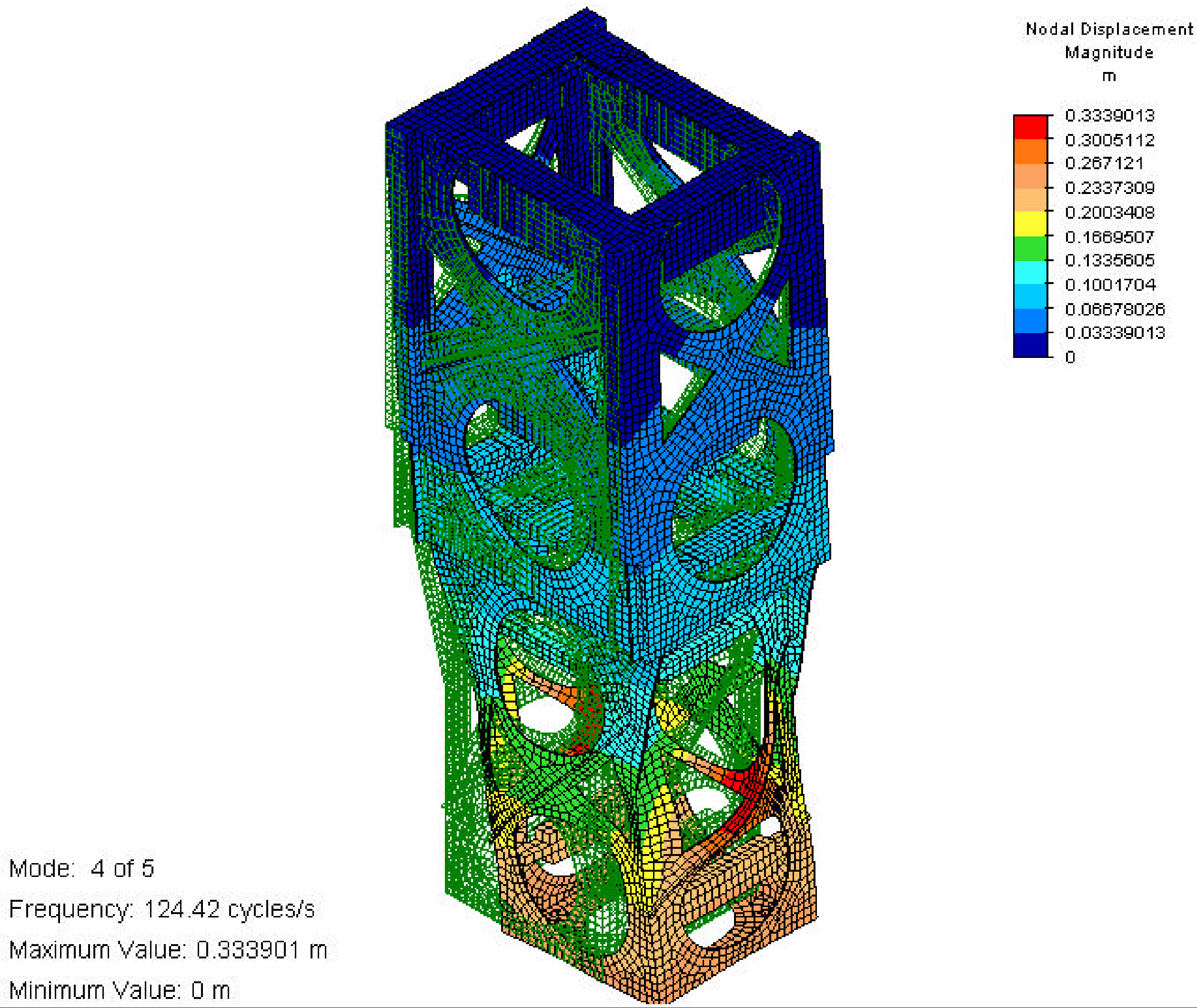
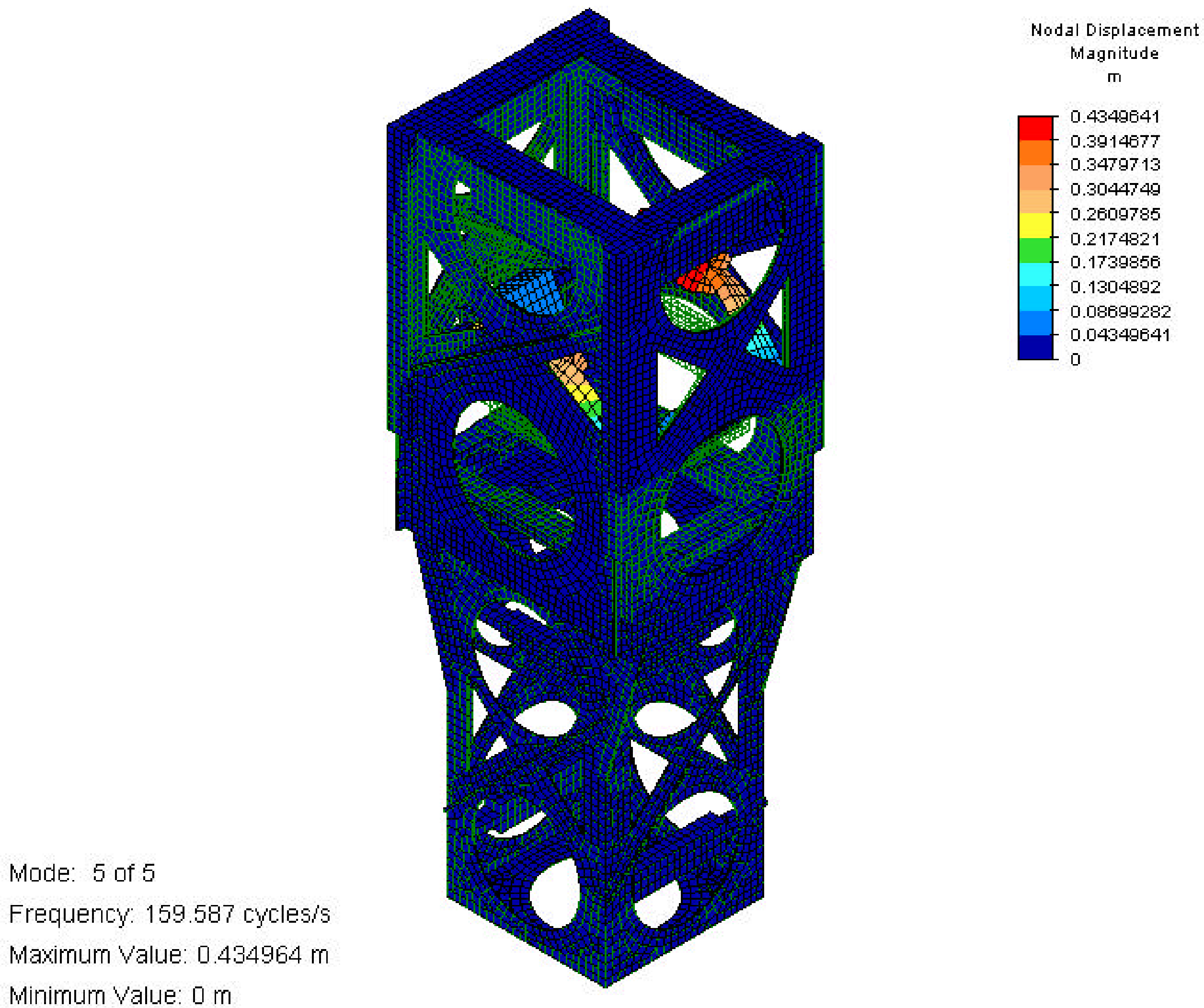


Figure 14: 5th mode, 160 Hz (local mode of nonstructural mass & supports)



6 Attachment Design

As noted above the assumption of a clamped boundary condition at the interface with the optics table is unrealistic unless the interface is designed properly. In particular, a few bar clamps at the perimeter held down with a few ¼-20 screws may not be sufficient; The entire quad structure will vibrate on these attachments “springs” and the “base” or “attachment” plate will bend between the attachment point/areas. In the analyses presented above the base plate was assumed to be clamped everywhere to the optics table.

Consider first extension of the attachment bolts. The number of bolts, n, required to have a minimum vertical bounce frequency of f is:

n = (l * m)/(E * A * gc) * (2 * pi * f)^2

where in english units:

E = 28e6 psi (steel)

l = 0.75 in (bolt length)

m = (60 kg structure + 70 kg nonstructural mass) * 2.205 = 287 lbm

gc = 386 lbm-in/lbf/s^2 (slinch)

f = 150 Hz

for a 1/4-20 bolt, the cross-sectional area, A = 0.027 in^2 and the number of bolts, n = 0.7. Consequently a single 1/4-20 bolt suffices to keep the vertical bounce mode above the 150 Hz requirement.

The above is the frequency for extension only. The "pendulum" mode with the suspended moment of inertia of the payload will be worse. Consider the number of bolts along the long side of the suspension structure, nl. The stiffness of the assembly in rotation about the opposite long edge will be approximately:

kr = nl * d^2 * k

where d is the short length of the suspension planform (550 mm) and k = E * A / l is the stiffness of an individual bolt. I have taken the axis of rotation along an edge rather than the center of the structure as a worse case. The above is approximate, and conservative, because I am not counting on the bolts along the side (since d^2 drops their contribution quickly).

The rotational frequency is then:

fr = (1/(2*pi) * SQRT(kr / Inertia)

where Inertia is the inertia of the quad structure alone (the optics and reaction chains do not participate) about the axis formed by a long edge of the structure at it's interface with the optics table. Solidworks gives me the following mass properties of the quad structure:

InertiaCG = 39 kg-m^2, about a parallel axis through the structure's cg

Zcg = -0.72 m, the height of the cg relative to the optics table

m = ~130 kg

so, Inertia = InertiaCG + m * (Zcg^2 + (d/2)^2) = 110 kg - m^2

nl = Inertia * l / (d^2 * E * A * gc) * (2 * pi * f)^2 = 1.8

So, two 1/4-20 bolts per side ~~is~~ may be adequate.

Base plate bending, due to inadequate attachment points, might also lower the frequency of the first natural mode. This is best addressed with a finite element analysis. This analysis is still pending.

7 Appendix: Limitations of Solid Elements for analysis of plate/shell structures

The analysis of the stiffened-shell structure in version –02 of this document was based on solid tetrahedral elements of second order (with no mid-side nodes) with only a single element through the thickness of the thin plates. While this approach is adequate to model the membrane stiffness of the plates, and hence the overall stiffness and dynamics of the structure acting as a complex beam, it is not adequate to model the bending stiffness of the plates. If the rib-stiffening of the plates were sufficient to push the plate bending modes well above the whole-structure bending modes (which was the intent), then this approximation should be adequate to model the first (whole structure, bending) mode. However, if the rib-stiffened plate bending modes are below the whole structure bending modes, the structure does not act as a unified body and the analysis is not accurate.

Comparison of first plate vibration frequency as calculated by theory and by different element formulations, different finite element modeling programs, and different mesh sizes are shown in Table 5. There are two obvious conclusions one can make from these comparisons:

- The linear tetrahedral solid element should never be used. Note that Algor *apparently* has higher order integration element options with the tetrahedral without employing mid-side nodes (unlike I-DEAS). For Algor, the mid-side node option should always be used.
- Since the solid elements do not have rotational degrees of freedom at their nodes, only free and clamped boundary condiitons can be approximated in the analysis and not simply supported. This is a typical limitation of solid finite element formulations.

The solid element mesh employed in version –02 of this document had ~17k elements. The convergence rate (as indicated in the results of Table 6 and the figure below) indicates that at least 50k solid elements (tetra and bricks) should be used in this structure to get close to converged results. Again a shell mesh for the plates (with either solid or beam elements for modeling the stiffeners) would be a better approximation.

Figure 15: Modal convergence with mesh size

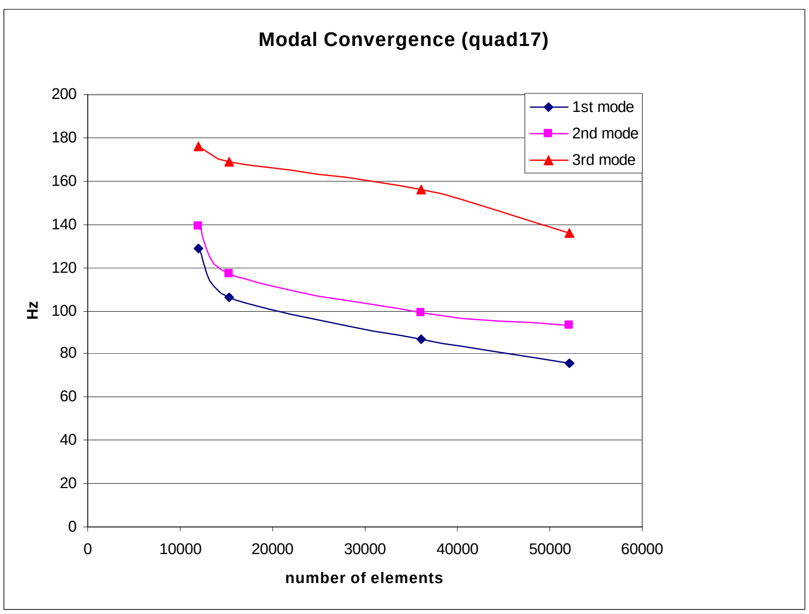


Table 5. Comparison of frequency calculations for the first plate bending mode

300 mm x 100 mm x 1 mm aluminum plate with simply supported short edges and varying boundary conditions on the long edges

					I-DEAS version 9					Algor version 13.32						Ansys version 8.0	
	Boundary Condition [0]	SS Pilkey [1]	SS Leissa [2]	CC Leissa [2]	4 node, linear quadrilateral, shell	8 node, parabolic quadrilateral, shell	linear, tetrahedral solid [3]	parabolic, tetrahedral solid [3]		plate	brick, solid [4]	tetrahedral solid: no mid-side nodes [5]		tetrahedral solid: mid-side nodes [5]		high order quad solid	
					140 elements	140 elements	2130	2148	22876	140	140	560	1788	560	1788	140	1200
1	FF	26	27	61	26	26	646	61	60	26	61	896	604	63	62	61	60
2	SF	-- [6]	61	89	58	58		131		58	138				133		
3	CF	--	114	137	108	109				109	138			176	133	128	126
4	CS	400	401	422	409	400				407	649				629		
5	CC	566	567	572	584	566		596		576	649			861	629	620	581
6	SS	270	270	284	274	274				274	649				629		

Notes:
[0] F = free, S = simply supported, C = clamped
[1] W. Pilkey, P. Chang, Modern Formulas for Statics and Dynamics, McGraw Hill, 1978.
[2] A. Leissa, Vibration of Plates, NASA SP-160, 1969.
[3] only one solid element through the thickness.
[4] Same results for with or without mid-side nodes, 2nd, 3rd and 4th order.
[5] Identical results with 2nd, 3rd and 4th order tetrahedral solid elements.
[6] The graphs and formulas in version -02 of this document based on Pilkey are not applicable for the SF case with an aspect ratio of 3:1

Table 6. Frequency convergence versus mesh size

				mass (kg)			1st mode		2nd mode		3rd mode		
FEM	**code**	**# elem**	**element type**	**non-struct**	**struct**	**total**	**freq (hz)**	**shape**	**freq (hz)**	**shape**	**freq (hz)**	**shape**	**comments**
quad14	Algor		brick & tetra, no mid-side nodes, 2nd order				109	y-bending	117	x-bending	197	local	
quad14	Algor		brick & tetra, mid-side nodes, 3rd order				84	y-bending	93	x-bending	171	local	
quad15	Algor	17745	brick & tetra, no mid-side nodes, 2nd order	70.0	51.0	121.0	117	y-bending	123	x-bending			used in T030044-02
quad16													given to Calum; post T030044-02 revision?
quad17	Algor	12027	brick & tetra, mid-side nodes, 3rd order				129	y-bending	139	x-bending	176	local	
quad17	Algor	15348	brick & tetra, mid-side nodes, 3rd order				106	y-bending	117	x-bending	169	local	non-structural mass & positions per
quad17	Algor	36129	brick & tetra, mid-side nodes, 3rd order				87	y-bending	99	x-bending	156	torsion	T030137-04
quad17	Algor	52117					76	y-bending	93	x-bending	136	torsion	structure length 2.005 m per N. Robertson
quad17	Ansys	50000	brick & tetra	65.0	51.2	116.2	72	y-bending & plate bending					error in access hole positions