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# Beam Deviation Due to De-centering of Power Recycling Mirror Curved Surface

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## 1. Introduction

The purpose of this note is to calculate the beam deviation that results from a de-centering of the curved surface of the power recycling mirror. This beam deviation will have an impact on the pre-alignment of the VIRGO interferometer. In addition, the allowed tolerance of the de-centering will have an impact on the global accuracy of the lateral placement of the power recycling mirror within the tower.

## 2. Power Recycling Mirror Design:

The following description is taken from VIR-NOT-LAS-1390-146, C. N. Man, 31/03/2000. In order to lower the level of aberration loss, we propose that the input face of the recycling mirror to being a convex surface, keeping the inside surface unchanged for the recycling cavity. Starting from the 4.9 mm waist of the mode cleaner we can now design an off-axis two-mirrors telescope of negligible aberration loss which yields a divergent beam incident on the recycling mirror. The recycling mirror has then a curvature of the order of 4 to 5 m (in fact 4.1 m allows to fit a telescope for Virgo which has the same size as the telescope for the CITF) as input face for anti-reflection coating and a plane face for the reflection. A first example of possible telescope (see internal doc of A&I) gives losses of the order of $10^{-5}$ (private comm. P.Hello).

The figure 2 shows the proposed case of the plane-convex mirror with a beam diverging from the telescope to Mrec and refocused par Mrec to make a waist of 19.8 mm matched to the optics of Virgo. This is the **solution adopted for Virgo** today.

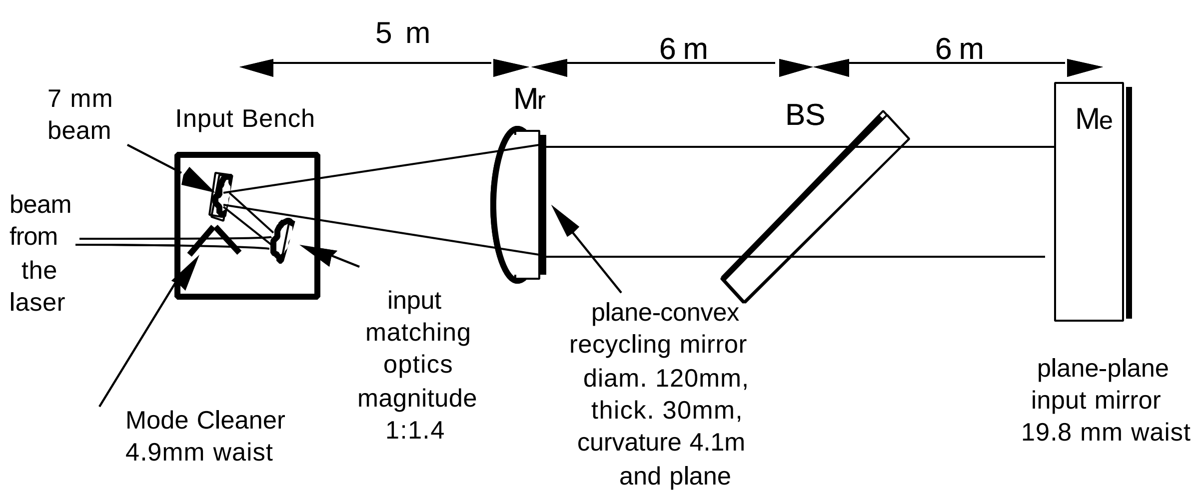


Figure 2: case of an unstable recycling cavity with a plane-convex recycling mirror.

## 3. Edge Thickness Error Due to De-centering of the Curved Surface

The nominal physical properties of the power recycling mirror are shown in the following:

| | |
|---|---|
| radius of curved surface, mm | $R := 4100$ |
| diameter, mm | $D_m := 120$ |
| center thickness, mm | $CT := 30$ |
| index of refraction, fused silica @ 1064nm | $n := 1.44963$ |

The edge thickness error can be determined using trigonometry, with reference to Figure 1: Geometry of the Power Recycling Mirror.

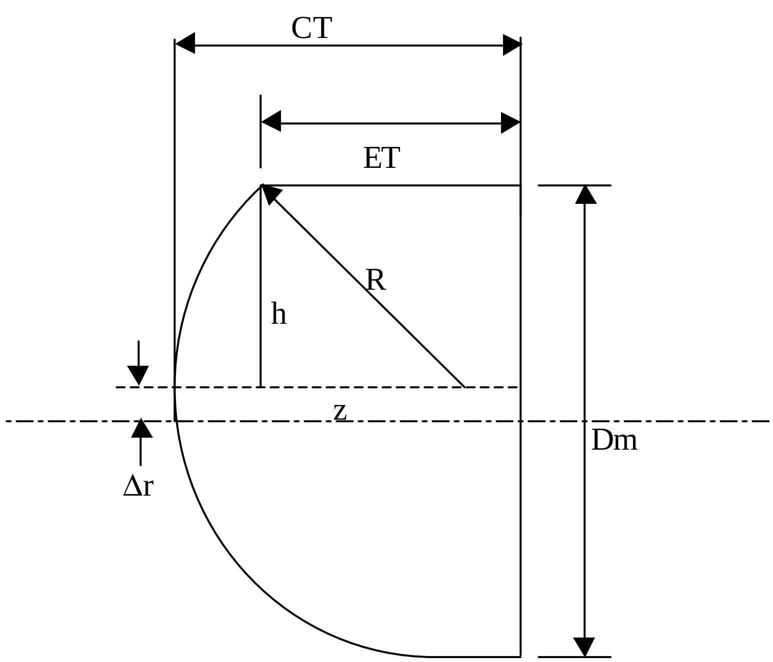


**Figure 1: Geometry of the Power Recycling Mirror**

The edge height, from the center of the curved surface to the upper edge of the mirror substrate, is determined by the mirror diameter and the de-centering distance.

$$h_{min}(\Delta r) := \frac{D_m}{2} - \Delta r$$

The corresponding edge thickness at the top of the substrate is determined by the mirror parameters and the edge height

$$ET_{max}(\Delta r) := CT - R + \left(R^2 - h_{min}(\Delta r)^2\right)^{0.5}$$

Likewise, the edge height from the center of the curved surface to the lower edge of the mirror substrate is given by

$$h_{max}(\Delta r) := \frac{D_m}{2} + \Delta r$$

And the corresponding edge thickness at the bottom of the substrate is given by

$$ET_{min}(\Delta r) := CT - R + \left(R^2 - h_{max}(\Delta r)^2\right)^{0.5}$$

The edge thickness error is the difference between the upper and lower edge thickness

$$\Delta_{ET}(\Delta r) := ET_{max}(\Delta r) - ET_{min}(\Delta r)$$

The edge thickness error as a function of the de-centering distance is shown in Figure 2: Edge Thickness Error Vs De-centering

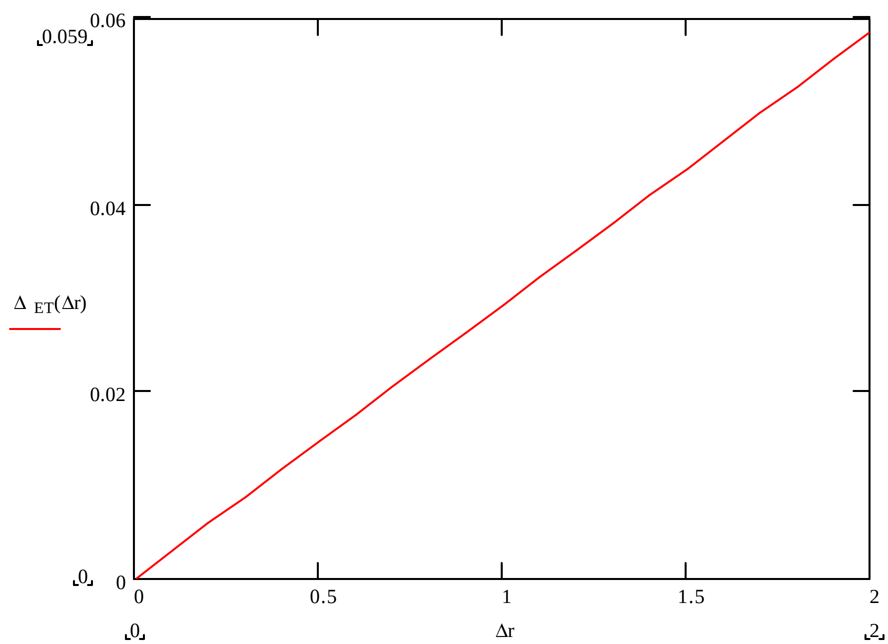


**Figure 2: Edge Thickness Error Vs De-centering**

Note that a 1mm de-centering will result in an edge thickness error of only 0.029mm.

## 4. Beam Deviation Due to De-centering of the Curved Surface

The power recycling mirror curved surface was chosen to provide an element of mode matching from the mode cleaner beam waist to the interferometer beam waist located at the input mirror. By propagating the 19.8 mm beam waist at the input mirror backwards through the power recycling mirror toward the mode cleaner, it can be determined that a virtual beam waist of 0.092 mm is located at a distance of 9119 mm upstream of the power recycling mirror in the direction of the mode cleaner. The angular deviation, due to displacing the input beam with respect to the axis of curvature of the power recycling mirror, can be modeled by propagating a displaced axial ray at the position of the virtual beam waist through the power recycling mirror to the input mirror location.

The ABCD system matrix, which includes the 9119mm translation from the virtual beam waist through the thick lens formed by the power recycling mirror followed by translation to the interferometer beam waist, was calculated to be the following

system matrix at IM $$M_{im} = \begin{pmatrix} -0.318 & 9.119\times 10^{3} \\ -1.097\times 10^{-4} & -6.924\times 10^{-6} \end{pmatrix}$$

Propagating an axial ray displaced by 1 mm causes an angular deviation of approximately 100 microradians, as shown below.

$$\Delta r := 1$$

input ray height at virtual waist, mm $$h_0(\Delta r) := \Delta r$$

input ray angle at virtual waist, rad $$\alpha_0 := 0$$

output ray height and angle at input mirror mm

$$h_{im}(\Delta r) := M_{im_{0,0}} \cdot h_0(\Delta r) + M_{im_{0,1}} \cdot \alpha_0 \qquad h_{im}(\Delta r) = -0.318$$

$$\alpha_{im}(\Delta r) := M_{im_{1,0}} \cdot h_0(\Delta r) + M_{im_{1,1}} \cdot \alpha_0 \qquad \alpha_{im}(\Delta r) = -1.097\times 10^{-4}$$

The beam deviation as a function of de-centering amount is shown in Figure 3: Beam Deviation Angle Vs De-centering.

## 5. Conclusions:

1. The edge thickness error for the power recycling mirror should be specified to be less than 0.03 mm, in order to limit the input beam deviation error to less than 100 microradians caused by the de-centering.
2. The centering of the power recycling mirror within the suspension tower should be within 1 mm of the true beam axis centerline, in order to limit the input beam deviation error caused by the de-centering.
3. The alignment of the input beam should be within 1 mm of the center of the power recycling mirror, in order to limit the input beam deviation error caused by the de-centering.
4. In the likelihood that the three conditions listed above can not be met simultaneously, the beam deviation caused by the de-centering errors will probably by much greater than 100 microradians, which will seriously hamper the pre-alignment of the interferometer.
5. The alignment procedure with the beam tube closed, described in VIR-PRO-CAS-1300-142: Pre-alignment of VIRGO Interferometer, will require that the beam deviation caused by the de-centering of the power recycling mirror be known and be correctable.
6. The alignment procedure with the beam tube open, described in VIR-PRO-CAS-1300-142: Pre-alignment of VIRGO Interferometer, is not affected by the unknown beam deviation caused by the de-centering of the power recycling mirror.

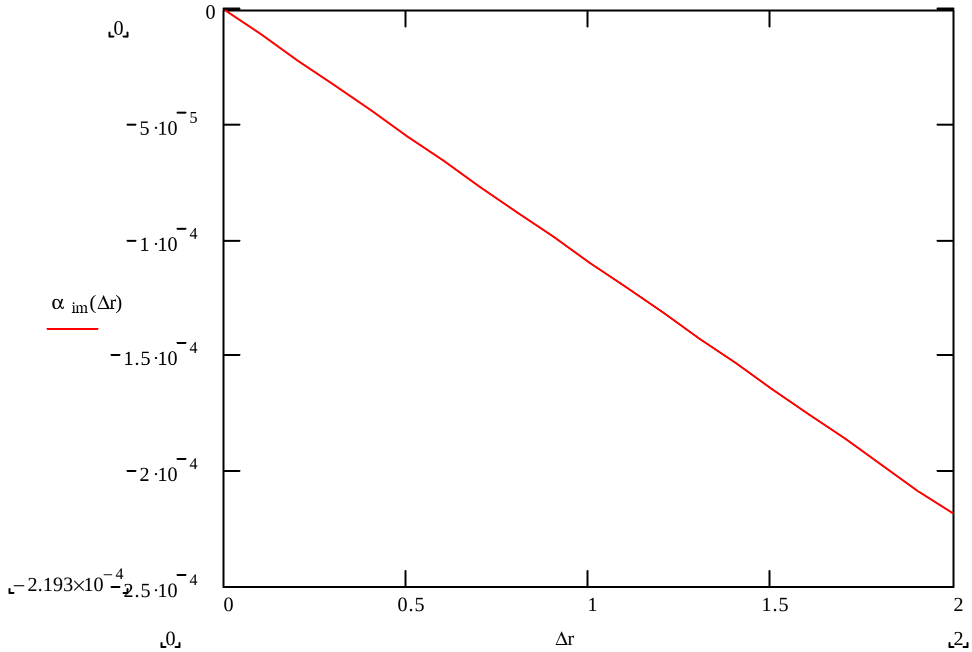


**Figure 3: Beam Deviation Angle Vs De-centering**