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Dumbbell Shaped Fibers for Advanced LIGO Suspensions

Phil Willems

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California Institute of Technology
LIGO Project - MS 51-33
Pasadena CA 91125
Phone (626) 395-2129
Fax (626) 304-9834
E-mail: info@ligo.caltech.edu

Massachusetts Institute of Technology
LIGO Project - MS 20B-145
Cambridge, MA 01239
Phone (617) 253-4824
Fax (617) 253-7014
E-mail: info@ligo.mit.edu

WWW: http://www.ligo.caltech.edu/

1 ABSTRACT

Dumbbell-shaped suspension fibers are proposed for the baseline design for advanced LIGO suspensions, as a relatively simple way to meet longitudinal thermal noise requirements and low vertical bounce frequency simultaneously. We present calculations of the noise spectrum and discuss other advantages such a geometry might have. We briefly discuss how such fibers may be fabricated and compare with twisted ribbon geometries.

2 KEYWORDS

suspensions, fused silica, thermal noise

3 INTRODUCTION

The choice between fused silica fibers and fused silica ribbons for advanced LIGO has, until now, been driven by the following tradeoff: ribbons can meet the proposed thermal noise requirements, but are difficult to make, while round fibers are relatively easy to make, but cannot meet the noise requirements.

The inability of fibers to meet the noise requirements specifically relates to the tradeoff between low longitudinal displacement noise and low vertical bounce frequency. If the length of the suspension and the suspended masses are taken as fixed, then the bounce frequency can be reduced only by reducing the fiber radius. However, there is an optimal fiber radius that minimizes longitudinal thermal noise, essentially by setting the strain to cancel the nonlinear thermoelastic damping. This optimal radius sets the vertical bounce frequency well above 10Hz.

Ribbons are, in theory, able to evade this tradeoff by making the ribbon width and thickness independently variable. The cross-section is set small enough for a suitably low vertical bounce frequency. This leads to a large static strain, and thus a high thermoelastic damping strength. The ribbon thickness is then made small enough (and the width correspondingly large) so that the dilution factor is large enough, and the thermoelastic peak frequency high enough, that the thermal noise at 10Hz meets requirements. The price paid for this flexibility is the difficulty of making ribbons. Very little experience in manufacture of high-Q ribbons exists, they are harder than fibers to make strong and precise, and they must have twists along their length to prevent buckling during pendulum oscillation- the suitability of these twists is yet untested.

At low frequencies, the longitudinal thermal noise of a fiber suspension is predominantly generated at the very ends of the fiber, where the bending is concentrated in pendulum motion. The dissipation in the middle section of the fiber contributes very little. The vertical bounce frequency, by contrast, samples the whole of the fiber equally. Therefore, we propose an improved fiber geometry, where the end regions have the optimal radius for longitudinal thermal noise, and the middle is made thinner to reduce the vertical bounce frequency- the dumbbell fiber.

This technical memo will first derive the equations of motion for a fiber made from several shorter, uniform fibers joined at their ends. It will then solve these equations and calculate the longitudinal thermal noise, and compare the calculations to those for a uniform fiber to show that the thinner middle regions do not increase the thermal noise at 10Hz. It will then derive equations for the vertical thermal noise and show that the vertical bounce frequency is reduced and the noise close to requirements. Lastly, we discuss some other potential merits of the dumbbell fiber and address some issues regarding their manufacture.

4 EQUATIONS OF MOTION FOR A DUMBBELL FIBER

The equations of motion for a dumbbell-shaped fiber are not very different from those for a uniform fiber. However, the equations must be solved for each segment independently and the solutions matched through boundary conditions. We have produced Mathematica code that solves the equations for fibers and ribbons and produces thermal noise spectra, entitled ‘dumbbell fiber.nb,’ ‘dumbbell test.nb,’ and ‘straight ribbon.nb.’ An outline of the theory is presented in the Appendix. In the notebooks, these numerical solutions are tested against known analytical solutions for straight fibers and against the approximation of Gretarsson et al., “Pendulum Mode Thermal Noise in Advanced Interferometers: A Comparison of Fused Silica Fibers and Ribbons in the Presence of Surface Loss.” The agreement is very good. (Note that the Gretarsson formula is updated to include nonlinear thermoelasticity.) The modelling of dissipation may be unfamiliar to those using bench, so we describe them briefly below.

5 MODEL FOR THE DISSIPATION

The dissipation in the fiber motion is different for bending motion and stretching motion. We point out that for a thin beam, bending is simply a stretching that varies linearly across the fiber, having a maximum tension at the outside of the bend and a maximum compression at the inside of the bend, with no stretching at the midpoint. We take a moment to clearly elaborate the loss mechanisms.

Intrinsic loss

This is just the loss of bulk fused silica independent of surface or thermoelastic effects. It is assumed frequency-independent with value $2\text{x}10^{-8}$, based on the most recent results from Syracuse. It enters equally into bending and stretching motion.

Thermoelastic loss

This is the loss due to thermoelastic damping, which arises when there is nonuniform strain in a material. As such, it enters into damping but not into stretching. The formula for the loss is

$$\phi_{TE} = \frac{E(\alpha - u_0\beta)^2 T}{\rho C_v} \frac{\omega\tau}{1+\omega^2\tau^2} \qquad t = \frac{4r^2}{13.55 D_{th}}$$

We ignore any longitudinal thermoelastic damping due to thermal gradients at the clamps or fiber boundary regions.

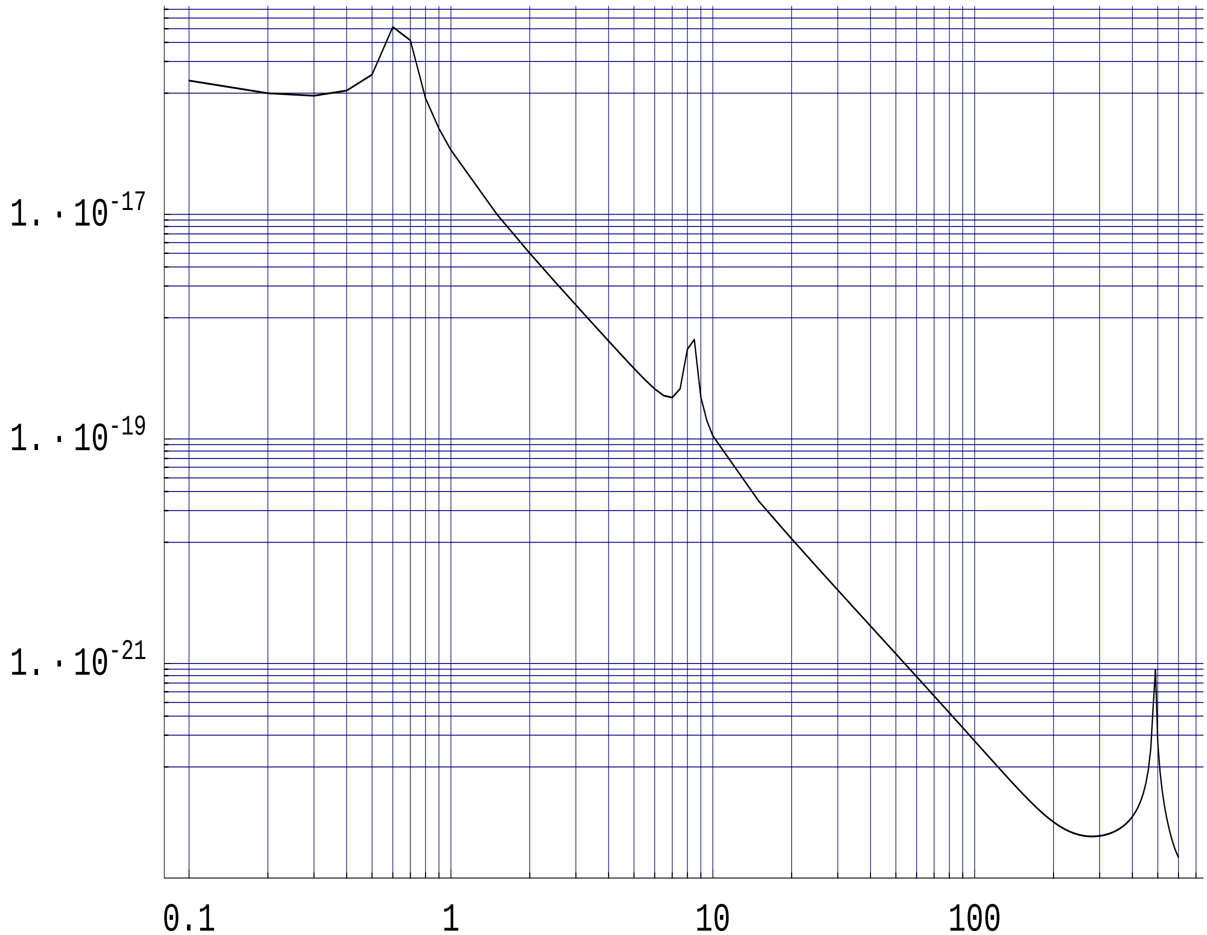
Surface losses

These are losses in the surface of the fiber as analyzed by Gretarsson et al. These will be present both in bending and in stretching; however, they will be more important in bending because bending concentrates the stress energy in the surface region, while stretching stresses the whole of the cross section equally. In the limit that the lossy surface is thin compared to the fiber itself, the difference is a factor of two; we assume this to be the case. For thin wide ribbons the factor is close to three. The value taken for the surface loss in bending is $(3 \times 10^{-11} m)/r$, consistent with the value used in bench, although a value of $(2 \times 10^{-11} m)/r$ is more consistent with violin modes Q's recently measured at Caltech. We also note that the same value of surface loss is divided by r or by the thickness of the ribbon t when comparing fibers to ribbons, as appears to be the way it is done in bench, although the ribbon damping should be 3/2 larger according to Gretarsson et al.

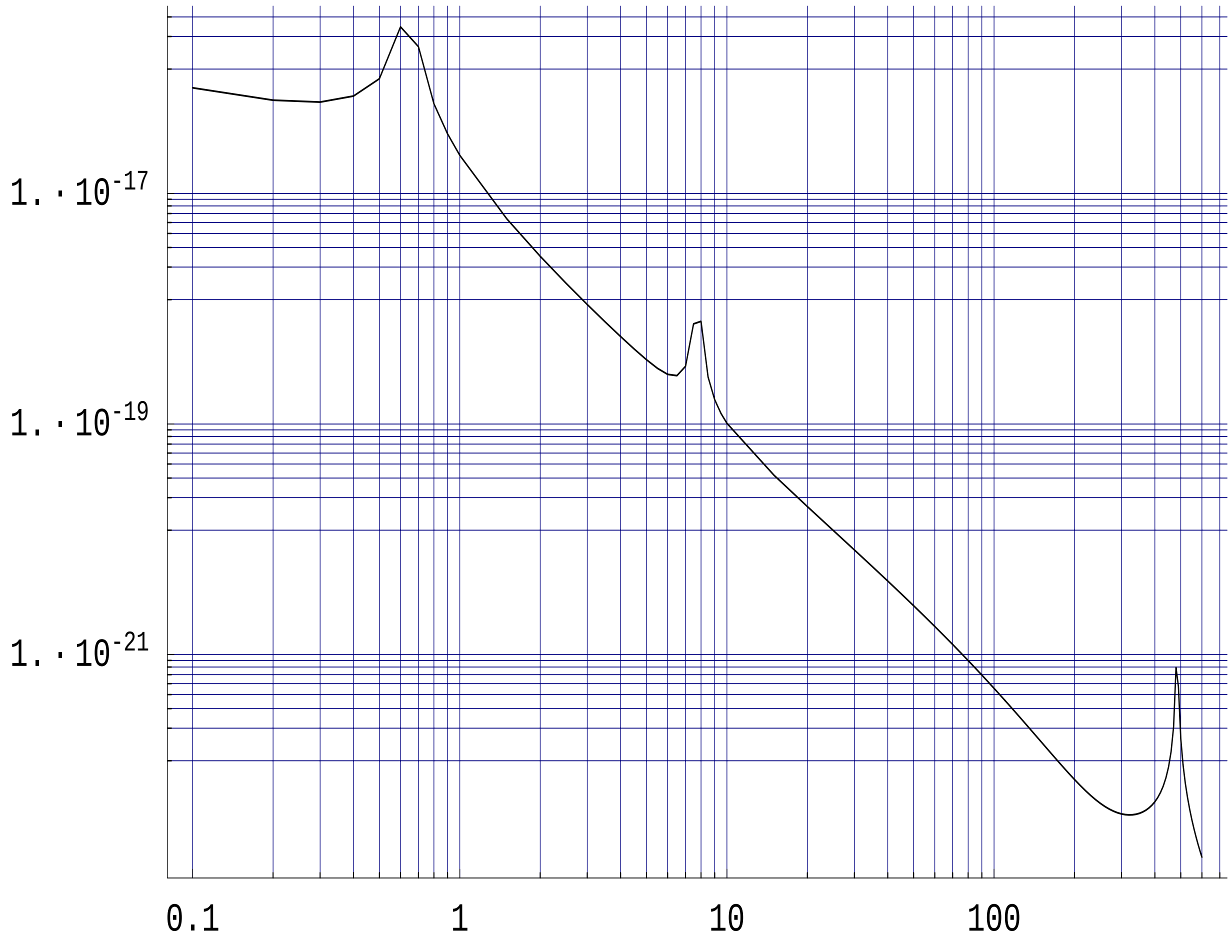
6 RESULTS OF THE MODEL

The formalism described in the last chapter was implemented using Mathematica to calculate the thermal noise spectrum for a fused silica fiber supporting 10kg, with a length of .6m, comparable to the advanced LIGO baseline suspension (where four fibers support 40kg). The ends of the fiber were given thicknesses of 767um, to have a stress that cancels the thermoelastic damping exactly. These ends were 10cm long, and the middle 40cm was given a thickness of 380um.

Here is the thermal noise spectrum calculated by the model for this dumbbell fiber:



Here is the thermal noise plot for the ribbon used in the advanced LIGO baseline model (note: the twists have not been modelled):



We see that both the dumbell fiber and the ribbon meet the thermal noise requirements at 10Hz. The fiber is slightly better at higher frequencies because it has no thermoelastic damping near the pendulum frequency.

7 DISCUSSION

The results presented show that dumbbell shaped fibers can meet advanced LIGO noise requirements, even though uniform fibers cannot. This gives advanced LIGO another option besides twisted ribbons in the suspension design.

The dumbell-shaped fiber is arguably superior to the twisted ribbon in several respects. This fiber is a fairly modest extension of technology already in hand, and could be made in several different ways: the center region can be drawn when the fiber is made or after it is installed. The strength of such a design is essentially already proven both by Caltech and by Glasgow. Ribbons have not yet been proven strong enough for advanced LIGO, and have not been tested with twists at all.

The quality factors of fiber suspensions are now in excess of 4×10^8, while those of ribbons are not yet proven superior, or even equal.

Given that they can meet the thermal noise requirements with relatively little risk, we suggest that dumbbell shaped fibers be adopted in the baseline design for advanced LIGO.

APPENDIX 1 EQUATIONS OF MOTION FOR A DUMBBELL FIBER

We assume the fiber to be made of three segments, the outer two having identical lengths and identical uniform diameters, the inner having a uniform diameter that is thinner. The transitions between the sections are abrupt. Each section is described by the dynamic beam equation:

$$EIX^{(iv)}(z,t) - TX''(z,t) = \rho\ddot{X}(z,t)$$

Here E is the Young's modulus, I is the bending moment of the fiber, T is the tension, and ρ is the linear mass density of the fiber. If we assume sinusoidal motion, for which

$$X(z,t) = X(z)e^{i\omega t}$$

then the general solution takes the form

$$X_i(z) = A_i\cos(k_{ti}z) + B_i\sin(k_{ti}z) + C_i\cosh(k_{ei}z) + D_i\sinh(k_{ei}z)$$

where i labels the fiber segment, and

$$k_{ti} = \sqrt{\frac{-T + \sqrt{T^2 + 4EI_i\rho_i\omega^2}}{2EI_i}} \qquad k_{ei} = \sqrt{\frac{T + \sqrt{T^2 + 4EI_i\rho_i\omega^2}}{2EI_i}}$$

and the A,B,C,D are constant coefficients determined by the boundary conditions. Notice that dissipation in the fiber is not modelled here. This is because the loss is assumed to be so small as to not perceptably influence the dynamics. This approximation fails very close to resonance but is quite good away from resonance.

For three fiber segments, there are 12 coefficients, requiring 12 boundary conditions. They are: at the top of the fiber, rigid clamping to fixed support

$$X_1(0) = X_1'(0) = 0$$

At the first joint between segments (z1),

$$X_1(z1) = X_2(z1)$$
$$X_1'(z1) = X_2'(z1)$$
$$I_1 X_1''(z1) = I_2 X_2''(z1)$$
$$EI_1 X_1'''(z1) - TX_1'(z1) = EI_2 X_2'''(z1) - TX_2'(z1)$$

These are, in order, conditions that the fiber is continuous, that its slope is continuous, that the moment is continuous, and that the force is continuous. At the second joint between segments (z2),

$$X_2(z2) = X_3(z2)$$
$$X_2'(z2) = X_3'(z2)$$
$$I_2 X_2''(z2) = I_3 X_3''(z2)$$
$$EI_2 X_2'''(z2) - TX_2'(z2) = EI_3 X_3'''(z2) - TX_3'(z2)$$

At the bottom of the fiber, an applied force, with the constraint that the fiber end not rotate,

$$EI_3 X_3'''(z3) - TX_3'(z3) = G \qquad X_3'(z3) = 0$$

This constraint against rotation approximates the four-fiber suspension design, in which pendulum and pitch motion are nearly decoupled.

These 12 equations in 12 unknowns are solved numerically, yielding the motion of the fiber in response to a force G at frequency ω. The dissipation W associated with this motion is estimated by multiplying the energy in bending 1/2EI(X'')^2 per unit length of fiber by 2π times the loss angle of the fiber as described in section 5, and integrating over the fiber length. The force at the fiber end equals the force required to accelerate the suspended mass plus some additional driving force. By subtracting the mass acceleration from G we obtain the driving force F, and thereby the ratio of the driving force to velocity of the mass, which is the admittance Y. Since the calculation of the fiber bending assumes no loss, the admittance is infinite at the fiber resonances, so the calculation is unreliable there.

The thermal noise is given by the fluctuation-dissipation theorem by

$$x^2(f) = \frac{k_B T}{\pi^2 f^2} Re[Y(f)]$$

The real part of the admittance can be evaluated by noting that the dissipation is given by

$$W = \int_0^{2\pi/\omega} Re[F]Re[v]dt = \int_0^{2\pi/\omega} Re[F]Re[FY]dt = Re[Y]\int_0^{2\pi/\omega} (Re[F])^2 dt$$

We are free to choose F to be real, and since we have already calculated the dissipation, we get the result

$$Re[Y] = \frac{W}{\pi F^2}$$

Finally, we must correct our model to account for the actual four-wire geometry. Since our model calculates the deflection of a single wire, the force required to deflect four wires that amount is four times larger, and the dissipation is also four times larger. Plugging these changes into the formula for Re[Y] thus reduces it by four. The vertical bounce motion of the system is simpler to model. The speed of sound for longitudinal waves in the fiber is ~6km/s, so for frequencies well below 6km/s/.6m=10kHz the system is well approximated by a mass on a spring, and the thermal noise spectrum is just

$$x^2(f) = \frac{4k_BTk\phi}{2\pi f\left[(k - m(2\pi f)^2)^2 + k^2\phi^2\right]}$$

The spring constant of the fiber is just the spring constants of its three sections combined in parallel; in the limit of small dissipation the real and imaginary parts are

$$k = \frac{k_1k_2}{k_1 + 2k_2} \qquad \phi = \frac{2k_2}{k_1 + 2k_2}\phi_1 + \frac{k_1}{k_1 + 2k_2}\phi_2$$

where k_1, k_2, ϕ_1, ϕ_2 are the spring constants and losses for the (1) end and (2) middle of the fiber. The spring constant for the i^{th} segment is

$$k_i = \frac{\pi r_i^2 E}{L_i}$$

The dissipation of a segment is discussed in section 5.