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# Update On The Flexure Design

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# Update on the Flexure Design

This document presents a brief update on the flexure design which utilises a thick wire with a threaded flare on either end to facilitate attachment to the cantilever spring/instrument pod. The spring and flexure are shown in figures 1 and 2 together with the attachments to the spring (the red and blue components). The attachment to the spring has changed and is now much simpler than in previous designs. The red riser is has a 5/8" thread on its inner surface and sits on top of the spring. The flexure is screwed into the riser until its shoulder is flush with the riser's upper surface (this defines the position of the upper pivot point). The flexure can then be screwed into the threaded hole in the instrument pod. Finally the riser/flexure is secured to the spring with the blue lock nut. There is NO adjustability in the design.

It appears that there is still some confusion (on my part) about the correct calculation to determine the position of the flexure's virtual pivot point. Stanford presented a calculation which showed the virtual pivot point of a cantilever beam of length L to be

$$V_p = \frac{1}{k}\tanh\left(\frac{kL}{2}\right) \qquad (10)$$

where $k^2 = P/EI$. P is the tension carried in the beam, E is the Young's Modulus and I is the second moment of area. I will denote this method #1. Appendix A presents a calculation (following the method of Eastman[1]) which determines the virtual pivot point of a cantilever beam whose free end is constrained to parallel translate. I will denote this method #2. Although both methods can give significant variations in the position of the virtual pivot point, both designs can be accommodated in the retrofit by some suitable alteration of parameters. As a result I will continue by considering both methods. Tables 1 and 2 give an overview of suitable parameters for methods #1 and #2 respectively.

M=mass hung from the flexure
$R_{flexure=}$radius of the flexure
L=total length of the flexure
$V_{p=}$position of the virtual pivot point
$L_{eff=}$distance between the pivot points (set by the current geometry)
$R_{sf=}$radius of the shoulder fillet
$\sigma_{load=}$stress from axial tension
$\sigma_{bending=}$stress from a 1mm translation
$K_{tension}$=stress concentration factor for axial tension
$K_{bending}$=stress concentration factor for bending

As method #1 results in virtual pivot points which are larger than those of method #2, the radius of the flexures have been reduced in order to fit the assembly into the 1.5" of headroom above the cantilever springs.

The stress concentration factors were estimated from expressions presented in Young[2]. Figure 3 shows stress concentration factors for the parameters shown in table 2.

If it is assumed that the effect of a shoulder fillet will result in some ambiguity in the position of the virtual pivot point then the radius should be as small as possible (i.e. the flexure will behave like a beam with a non-uniform cross-section at the clamping point. This could be modeled with an FEA program?). Therefore the fillet radius was chosen to be 1mm. This gives a good compromise between pivot point ambiguity and stress concentration factors. For example, increasing the radius to 2mm doesn't help too much with stress concentration but will make the pivot point ambiguity worse. It is possible that the radius could be reduced to 0.5mm, although the total stress starts to get uncomfortably close to the fatigue stress of maraging 300 (which has an asymptotic value of $8.5x10^8 N/m^2$)

| Stage #1 | Stage #2 |
|---|---|
| M=208kg | M=134kg |
| $R_{flexure}$=1.8mm | $R_{flexure}$=1.5mm |
| L=0.141m | L=0.134m |
| $V_p$=27.6mm | $V_p$=24mm |
| $L_{eff}$=85.9mm | $L_{eff}$=85.9mm |
| $R_{sf}$=1mm | $R_{sf}$=1mm |
| $\sigma_{tension}=2x10^8 N/m^2$ | $\sigma_{tension}=1.9x10^8 N/m^2$ |
| $\sigma_{bending}=1.1x10^8 N/m^2$ | $\sigma_{bending}=1.1x10^8 N/m^2$ |
| $K_{tension}$=1.6 | $K_{tension}$=1.5 |
| $K_{bending}$=1.4 | $K_{bending}$=1.3 |

Table 1. Parameters for method #1.

| Stage #1 | Stage #2 |
|---|---|
| M=208kg | M=134kg |
| $R_{flexure}$=2mm | $R_{flexure}$=2mm |
| L=0.123m | L=0.125m |
| $V_p$=18.6mm | $V_p$=19.5mm |
| $L_{eff}$=85.9mm | $L_{eff}$=85.9mm |
| $R_{sf}$=1mm | $R_{sf}$=1mm |
| $\sigma_{tension}=1.6x10^8 N/m^2$ | $\sigma_{tension}=1.0x10^8 N/m^2$ |
| $\sigma_{bending}=1.8x10^8 N/m^2$ | $\sigma_{bending}=1.7x10^8 N/m^2$ |
| $K_{tension}$=1.6 | $K_{tension}$=1.6 |
| $K_{bending}$=1.4 | $K_{bending}$=1.4 |

Table 2. Parameters for method #2.

1 Eastman. F.S., 1935, Flexure pivots to replace knife edges and ball bearings, Bull. Univ. Washington Eng. Station No. 86; see also 1937 J. Aeronaut. Sci. **5** 16-21

2 Young, W.C., Roark's Formulas for Stress and Strain, 6th edition

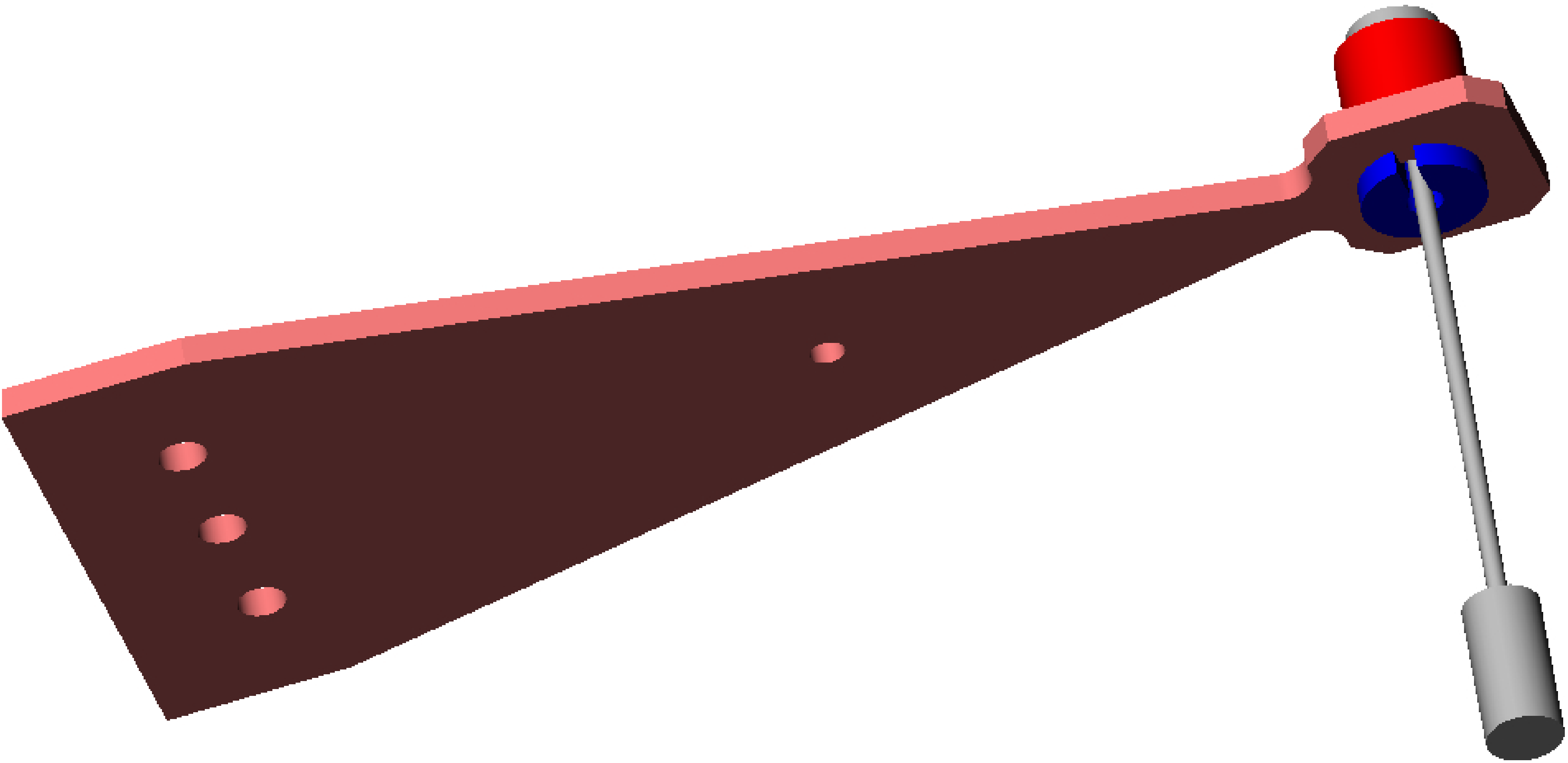

Figure 1. Rendering of the spring/flexure assembly

Figure 2. Section view of the spring/flexure assembly

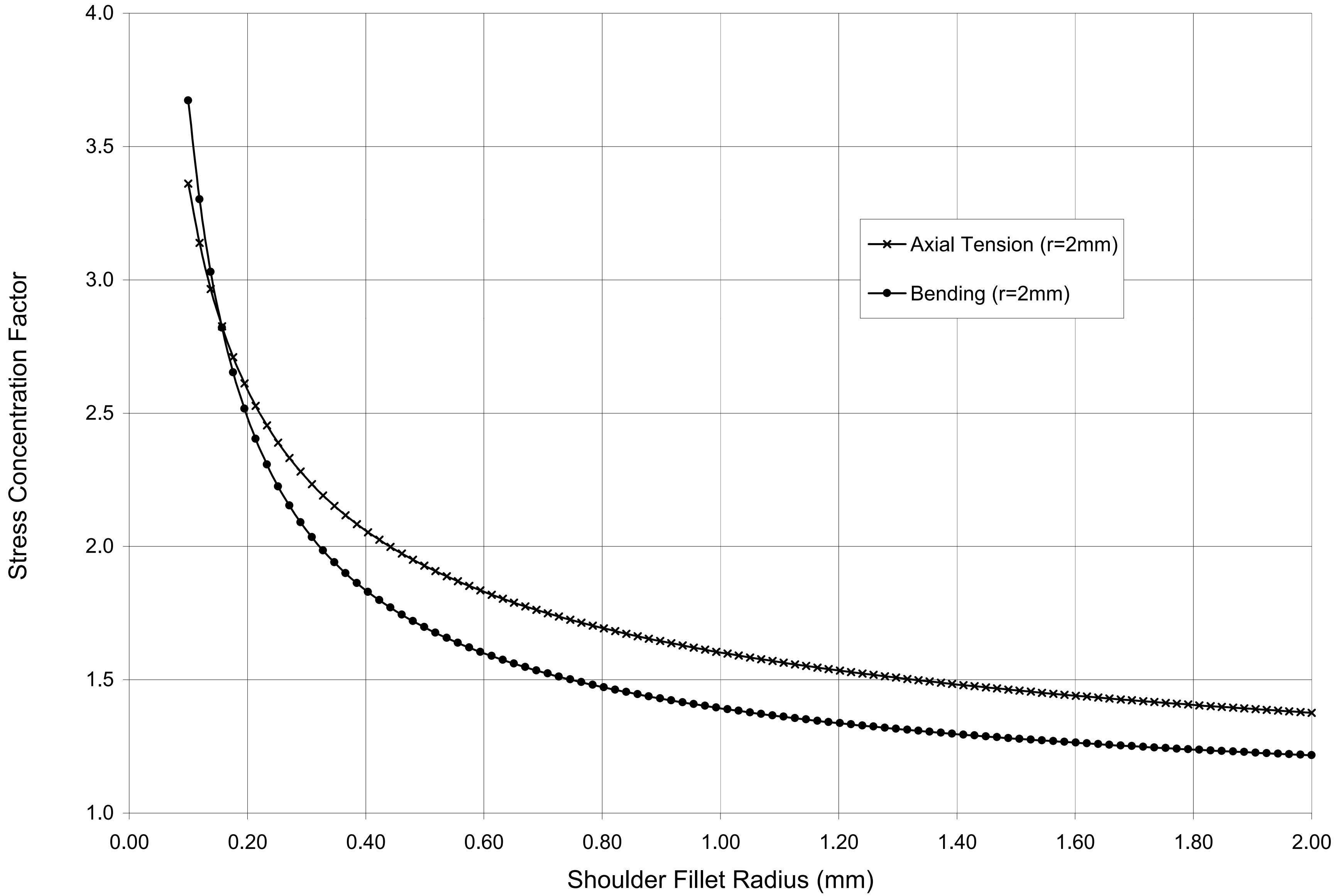


Figure 3. Stress concentration factors

# Appendix A: Virtual Pivot Point of a Cantilever Beam which is Constrained to Parallel Translate

Consider the cantilever beam shown in figure A1. In this derivation L is the length of the beam, E is the Young's modulus, I is the second moment of area ($\pi R^4/4$ for a circular cross-section of radius R), P is the tension and W is the transverse force. A virtual pivot point exists at a distance $V_p$ from either end of the beam.

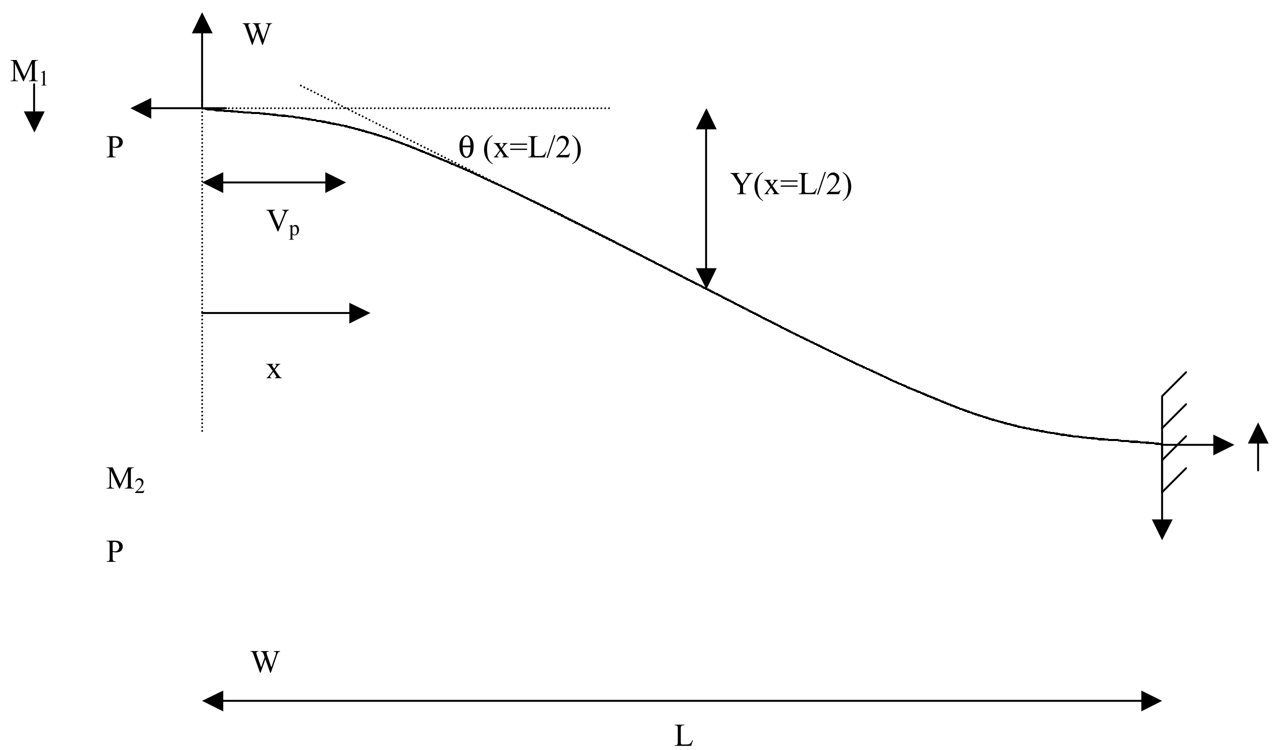


Figure A1. Geometry of the beam

The right end of the beam is fixed while the left end is allowed to parallel translate. This results in moments $M_1$ and $M_2$ acting on either end. The moment acting to the right of a point x is given by,

$$M = M_2 + Py - W(L - x) \qquad (1)$$

Differentiating twice with respect to x gives

$$\frac{d^2M}{dx^2} = P\frac{d^2y}{dx^2} = \frac{MP}{EI} = k^2M \qquad (2)$$

where $k^2 = P/EI$. The general solution to this equation is in terms of hyperbolic sines and cosines

$$M = C_1 \cosh(kx) + C_2 \sinh(kx) \qquad (3)$$

Utilising the boundary conditions M=$M_1$ at x=0 and M=$M_2$ at x=L gives, after some algebra

$$M = M_1 \cosh(kx) + \left[\frac{M_2 - M_1 \cosh(kL)}{\sinh(kL)}\right]\sinh(kx) \qquad (4)$$

The deflection of the beam as a function of x can then be obtained by substituting equation (4) into equation (1)

$$y = \left(\left[\frac{M_2 - M_1 \cosh(kL)}{\sinh(kL)}\right]\sinh(kx) + M_1 \cosh(kx) - M_2 + W(L-x)\right)/P$$

(5)

and the angle of the beam is then given by $dy/dx$

$$\theta = \left(\left[\frac{M_2 - M_1 \cosh(kL)}{\sinh(kL)}\right]k\cosh kx + M_1 k \sinh(kx) - W\right)/P \qquad (6)$$

Applying the boundary conditions θ=0 at x=0 and θ=0 at x=L gives, after some manipulation

$$M_1 = -\frac{S\sinh(kL)}{k[1+\cosh(kL)]}, M_2 = \frac{S\sinh(kL)}{k[1+\cosh(kL)]} \qquad (7)$$

The position of the virtual pivot point can be obtained by calculating y(x=L/2) and θ(x=L/2)

$$V_p = \frac{L}{2} - \frac{y(x=L/2)}{\theta(x=L/2)} \qquad (8)$$

where small angle approximations have been made. Finally, the bending stress in the beam can be obtained from

$$\sigma_{bend} = \frac{MR}{I} \qquad (9)$$

With R=1.5mm, E=$2x10^{11}$N/$m^2$, L=0.1175m, P=$1.98x10^3$N and S=25.6N the position of the virtual pivot point is 15.3mm from the end of the beam. Plots for the deflection, angle and bending stress are shown on the following pages. For comparison, the position of the virtual pivot point for a cantilever beam which is allowed to freely rotate is given by

$$V_p = \frac{1}{k}\tanh\left(\frac{kL}{2}\right) \tag{10}$$

which is 19.9mm using the numbers above.