Bill Kells 9/2/01
CONTINUATION TO "PARAMETRIC INSTABILITY IN FABRY-PEROT INTERFEROMETER"
(1)

T010098-00-D Braginsky / Strigin / Vyatchanin - Appendix

CONSIDER EXPLICITLY THE ANTI-STOKES MODE $\omega_2 \approx \omega_0 + \omega_m$
IN SAME NOTATION:

$$\partial_t D_2 + \delta_2 D_2 = \frac{i B_2 X D_0 \omega_0}{L} e^{-\Delta_2 t} \qquad \Delta_2 \equiv \omega_0 - \omega_2 + \omega_m \quad (A1')$$

FOLLOWING FROM

$$\mathcal{L}_{INT} = - \int \frac{\chi u_x \langle H_0 + H_2 \rangle^2}{8\pi} \Big|_{z=0} dr_\perp = -2\omega_0\omega_2 q_0 q_2 B_2 \frac{\chi}{L}$$

THE SOLUTION TO $(A1')$ AND THE ANALOG TO $(A2)$:

$$\partial_t X + \delta_m X = \frac{i B_2 D_0^* D_2 \omega_2 \omega_0}{M \omega_m L} e^{i\Delta_2 t} \qquad (A2')$$

IS THEN

$$D_2 (\delta_2 - i\Omega) = \frac{i B_2 D_0 \omega_0}{L} + \frac{i B_2 D_2 D_0^* \omega_2 \omega_1}{M \omega_m L (\delta_m - i\Omega)}$$

WHICH INDEED HAS only DAMPED SOLUTIONS

HOWEVER $(A1')$ INDICATES THAT D_2 will INEVITABLY BE EXCITED SO IT OUGHT TO BE RETAINED, WITH FULL INTERACTION:

$$\mathcal{L}_{INT} = - \int \frac{\chi u_x \langle H_0 + H_1 + H_2 \rangle^2}{8\pi} \Big|_{z=0} dr_\perp = -2\omega_0 q_0 B \frac{\chi}{L} (\omega_1 q_1 + \omega_2 q_2)$$

WHERE $B_1 = B_2 \equiv B$ IS ASSUMED TO HOLD ACCURATELY FOR LONG LIGO-like CAVITIES. (Ignore the $q_1 q_2$ term)

SOLUTIONS TO THE FULL SYSTEM:

$$\partial_t D_1 + \delta D_1 = \frac{iB\omega_0}{L} X^* D_0 e^{-i\Delta_1 t} \quad (A)$$

$$\partial_t D_2 + \delta D_2 = \frac{iB\omega_0}{L} X D_0 e^{-i\Delta_2 t} \quad (B)$$

$$\partial_t X + \delta_m X = \frac{iB\omega_0}{ML\omega_m} \left\{ D_0 D_1^* \omega_1 e^{-i\Delta_1 t} + D_0^* D_2 \omega_2 e^{i\Delta_2 t} \right\} \quad (C)$$

AGAIN, THE closely SPACED CAVITY MODES ALLOW $\delta_1 = \delta_2 \equiv \delta$

TAKE $\Delta_1 = \Delta_2 = 0$ (zero detuning). SEEK SOLUTIONS $X \equiv \chi e^{-i\Omega t}$

WHICH, SUBSTITUTED IN (A) & (B), GIVE:

$$D_1 = \frac{iB\omega_0}{L(\delta + i\Omega^*)} \chi^* e^{i\Omega^* t} D_0$$

$$D_2 = \frac{iB\omega_0}{L(\delta - i\Omega)} \chi e^{-i\Omega t} D_0$$

WHICH, IN TURN SUBSTITUTED INTO (C), GIVE:

$$\chi(\delta_m - i\Omega) = \frac{R_0 \Lambda \, \delta \delta_m}{\omega_0 (\delta - i\Omega)} \left\{ \chi \omega_1 e^{-i\Omega t} - \chi \omega_2 e^{-i\Omega t} \right\} e^{i\Omega t}$$

WHICH HAS INSTABILITY THRESHOLD, USING $\omega_2 - \omega_1 = 2\omega_m$:

$$2 R_0 \Lambda \frac{\omega_m}{\omega_0} > 1$$

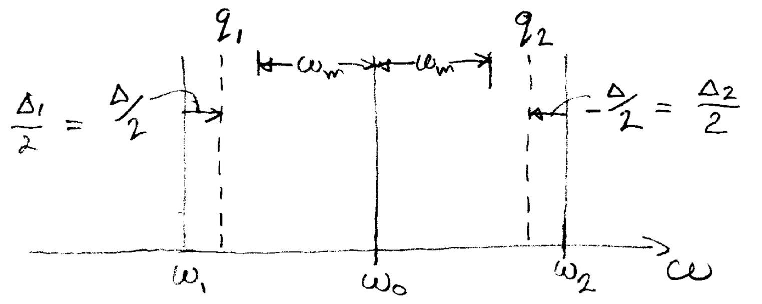
However $\omega_m / \omega_0 \sim 10^{-10}$

But is THIS "cancellation" AN ARTIFACT OF ZERO DETUNING?

AGAIN follow de tuned NOTATION LEADING TO (A5):

$$D_1(t) = D_1 e^{\lambda_- t}, \quad X^*(t) = \chi^* e^{\lambda_+ t} \quad D_2(t) = D_2 e^{\lambda_+ t}$$

NOTICE THAT THE DETUNING IS SYMMETRIC WRT. ω_0:



$\Delta_1 = -\Delta_2 \equiv \Delta$

THESE CHOICES LEAD TO THE CHARACTERISTIC EQUATION:

$$(\delta + \lambda_+)(\delta_m + \lambda_-) = A \frac{\omega_1 - \omega_2}{\omega_1} \approx 2A \frac{\omega_m}{\omega_0}$$

WHICH IS THE SAME FORM AS (A5)-(A9) BUT WITH

$$A \to A \frac{2\omega_m}{\omega_0} \sim 10^{-10} A$$

SO THE INSTABILITY IS SUPPRESSED, AS FOR THE ZERO DETUNE CASE.