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Characterization of the Pre-modecleaner for the Pre-stabilized Laser on the Hanford 4-km Interferometer

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The goal of this effort is to assess the internal losses in the pre-modecleaner (PMC) utilized in the LIGO pre-stabilized laser system (PSL). We use design parameters for the dimensions of the pre-modecleaner cavity and laboratory measurements (cavity bandwidth, input power, transmitted powers, and reflected power) to estimate the mirror transmissions and losses due to absorption and scattering. We are unable to determine the losses for each mirror, so only the total cavity loss and average loss per mirror are given. We assume that the transmissions of the two flat mirrors are equal.

FSR and Finesse

The PMC is a high-aspect-ratio, triangular Fabry-Perot cavity. If the base of the triangular optical path is denoted by d and the height by h, the optical path length, L, is given by,

$$L = d + \sqrt{d^2 + 4h^2}. \tag{1}$$

The free spectral range (FSR) of the cavity is given by

$$f_{\rm fsr} = \frac{c}{L} \tag{2}$$

where c is the speed of light. The finesse, $\mathcal{F}$, is defined as the ratio of the free spectral range to the width (FWHM) of the cavity resonance peak,

$$\mathcal{F} = \frac{f_{\rm fsr}}{\Delta f}, \tag{3}$$

where Δf is the width of the resonance (power, measured in transmission or reflection). The design parameters and calculated values are given in Table 1, below.

Table 1: PMC Parameters

parameter	symbol	num. value
height (cm)	h	20
base (cm)	d	2
path length (cm)	L	42.1
free spectral range (MHz)	f_{fsr}	713.0

Mirror Reflectivities

With the amplitude reflectivity and transmissivity of the cavity mirrors given by r_i and t_i for $i = 1, 2, 3$, the corresponding power reflectivity and transmissivity are given by

$$R_i = r_i^2, \tag{4}$$
$$T_i = t_i^2. \tag{5}$$

These are not entirely independent quantities. They are related by energy conservation;

$$R_i + T_i + \mathcal{L}_i = 1, \tag{6}$$

where $\mathcal{L}_i$ are the mirror power losses due to scattering and absorption. In what follows we assume that T_i and $\mathcal{L}_i$ are independent variables, and determine R_i from energy conservation.

A useful quantity for these calculations is the round-trip field reflectivity, r, given by

$$r = r_1 r_2 r_3. \tag{7}$$

The quantity which is directly related to the mirror reflectivities is the coefficient of finesse, F, given by

$$F = \frac{4r}{(1-r)^2}. \tag{8}$$

The coefficient of finesse, F, and the finesse, $\mathcal{F}$, are related by

$$\mathcal{F} = \frac{\pi\sqrt{F}}{2}. \tag{9}$$

Power Budget

The power of the laser beam incident on PMC is denoted by P_{las}. The incident laser power that is in the fundamental mode (TEM_{00}in this case) of the PMC, the mode-matched power, is given by is

$$P_{\text{mm}} = MP_{\text{las}}, \tag{10}$$

where M is the mode-matching factor.

The laser power circulating in the cavity is given by

$$P = GP_{\text{mm}} = GMP_{\text{las}}, \tag{11}$$

where G is the cavity amplification factor or power gain given by

$$G = \frac{T_1}{(1-r)^2}. \tag{12}$$

The subscript *1* denotes the input mirror of the cavity. Strictly speaking, the power in the cavity is different after reflection from the different cavity mirrors. P given above corresponds to the laser field immediately after the cavity input mirror. The powers inside the cavity immediately after the second and third mirrors are given by R_2P and R_3R_2P, respectively.

Thus the power transmitted (leaked) through the back mirror (the curved mirror at the vertex of the triangle between the two flat mirrors, mirror 3) is

$$P_{\text{leak}} = T_3R_2P. \tag{13}$$

The power transmitted through the cavity on resonance is given by

$$P_{\text{tr}} = T_2P. \tag{14}$$

Therefore the cavity throughput, the ratio of transmitted to incident power, is given by

$$T_{\text{cav}} \equiv \frac{P_{\text{tr}}}{P_{\text{las}}} = \frac{T_1T_2M}{(1-r)^2}. \tag{15}$$

Visibility and Mode-matching

The power reflected by the cavity is maximum when the cavity is anti-resonant (out of lock). In this case, for a reasonably high finesse cavity, it reflects almost all of the incident power,

$$P_{\text{max}} \approx P_{\text{las}}. \tag{16}$$

The minimum power reflected by the cavity (when it is in lock) is P_{min}. When the power in the sidebands used to lock the cavity is negligible (as it is for the PMC in the PSL), then

$$P_{\mathrm{min}} \approx P_{\mathrm{ref}} \tag{17}$$

where P_{ref} is the power reflected by the cavity when it is locked. P_{las}, P_{ref}, P_{tr},and P_{leak} are measured in the laboratory.

To calculate the cavity reflectivity when on resonance , we separate the mode-matched and non-mode-matched powers. The mode-matched and non-mode-matched powers are given by MP_{las}, and $(1-M)P_{\mathrm{las}}$, respectively. The total power reflected by the cavity can be written as

$$P_{\mathrm{ref}} = R_{\mathrm{cav}} M P_{\mathrm{las}} + R_1 (1-M) P_{\mathrm{las}}. \tag{18}$$

where $R_{\mathrm{cav}} \equiv |\rho|^2$ is the reflectivity of the cavity for perfectly mode-matched power. ρ is given by

$$\rho = r_1 - \frac{t_1^2 r_2 r_3}{1 - r_1 r_2 r_3}. \tag{19}$$

The cavity visibility is defined as

$$V \equiv \frac{P_{\mathrm{max}} - P_{\mathrm{min}}}{P_{\mathrm{max}}}. \tag{20}$$

It is related to the mode-matching factor as

$$V = M\left(1 - \frac{R_{\mathrm{cav}}}{R_1}\right). \tag{21}$$

Measurements

P_{las}, P_{tr}, andP_{ref}were measured using a Scientech calorimeter and P_{leak} was measured using an Ophir power meter. The cavity bandwidth was measured by sweeping the laser frequency using the FAST actuator and monitoring either the transmitted or reflected power. The parameters measured in March, 2001 are given in Table 2.

Calculation of Mirror Transmissions and Losses

Using the measured power levels and cavity bandwidth, an iterative calculation is used to estimate the mirror transmissions and losses. First a a guess

Table 2: Measured Parameters

parameter	symbol	num. value
input laser power (W)	$P_{\rm las}$	9.3
transmitted power (W)	$P_{\rm tr}$	6.13
reflected power (W)	$P_{\rm ref}$	1.35
leakage power (W)	$P_{\rm leak}$	0.11
cavity bandwidth (MHz)	Δf	4.33

of the mode-matching factor (the visibility, V, is a good first guess) is used to calculate the mirror transmissions and losses. Then these parameters are used to obtain a better estimate of the mode-matching factor which is then used to improve the estimate of the transmissions and losses, and so on.

First we calculate r using the measured cavity bandwidth and Eqs. (8) and (9). To start the iterative calculation, we provide an initial guess for the mode-matching factor, M. The T_1T_2 product is determined from the incident and transmitted power measurements using

$$T_{\rm cav} \equiv \frac{P_{\rm tr}}{P_{\rm las}} = MGT_2 = \frac{T_1T_2M}{(1-r)^2}. \tag{22}$$

and is given by

$$T_1T_2 = \frac{T_{\rm cav}}{M}(1-r)^2. \tag{23}$$

Assuming that the mirrors have equal transmissions we obtain

$$T_1 = T_2 = \sqrt{\frac{T_{\rm cav}}{M}}(1-r). \tag{24}$$

The transmission of the third mirror can be found from the leakage power by

$$T_3 = \frac{T_{\rm leak}}{T_1M}(1-r)^2. \tag{25}$$

The average mirror losses are estimated as follows. Consider the product,

$$\begin{aligned} R &\equiv R_1R_2R_3 && (26)\\ &= (1-\mathcal{L}_1-T_1)(1-\mathcal{L}_2-T_2)(1-\mathcal{L}_3-T_3) && (27)\\ &= 1-\sum_i \mathcal{L}_i - \sum_i T_i + \sum_{i<j} T_iT_j && (28)\\ &\quad + \sum_{i<j} L_iL_+ \sum_{i\leq j} L_iT_+ \text{higher order terms} && (29)\end{aligned}$$

Neglecting terms with $\mathcal{L}_i T_j$, $\mathcal{L}_i \mathcal{L}_j$, and higher order terms, we have

$$R \approx 1 - \sum_i \mathcal{L}_i - \sum_i T_i + \sum_{i<j} T_i T_j. \tag{30}$$

The combined losses are then given by

$$\sum_i \mathcal{L}_i = 1 - R - \sum_i T_i + \sum_{i<j} T_i T_j. \tag{31}$$

Assuming that the losses per mirror are equal, i.e. $\mathcal{L}_1 = \mathcal{L}_2 = \mathcal{L}_3$, we obtain an estimate of the average loss per mirror,

$$\mathcal{L} = \frac{1}{3} \sum_i \mathcal{L}_i. \tag{32}$$

Using this result, we obtain a correction for the combined losses by considering the fifth and sixth terms in the last line of Eq. 26,

$$\mathrm{corr} = 3\mathcal{L}^2 + 2\mathcal{L}(T_1 + T_2 + T_3), \tag{33}$$

Thus, a better estimate of the average loss per mirror is

$$\mathcal{L} \Rightarrow \mathcal{L} + \frac{1}{3}\mathrm{corr}. \tag{34}$$

Using the estimates of mirror transmissions and losses, we now calculate a better estimate of the mode-matching factor using the measured visibility by

$$M = \frac{V}{1 - R_{\mathrm{cav}}/R_1} \tag{35}$$

This value is then used to improve the estimates of the mirror transmissions and losses. This procedure is iterated until the estimates converge.

The results of the calculations are shown in Table 3.

Table 3: Calculated Parameters

parameter	symbol	num. value
cavity visibility	V	0.85
mode-matching coefficient	M	0.87
transmission of mirror # 1 and # 2	T_1	1.65×10^{-2}
transmission of mirror # 3	T_3	2.95×10^{-4}
average losses per mirror	$\mathcal{L}$	1.54×10^{-3}