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# Measurement of the Dependence of the Young's Modulus of Fused Silica on Temperature

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# 1 ABSTRACT

We have measured the temperature dependence of the Young’s modulus of Suprasil fused silica to be $(dE/dT)/E=+1.52\times10^{-4}/K \pm 5\%$. This sets the optimum stress of the suspension fibers with respect to the nonlinear thermoelastic effect to be approximately 240MPa.

# 2 KEYWORDS

Nonlinear thermoelasticity, fused silica, Young’s modulus, temperature.

# 3 INTRODUCTION

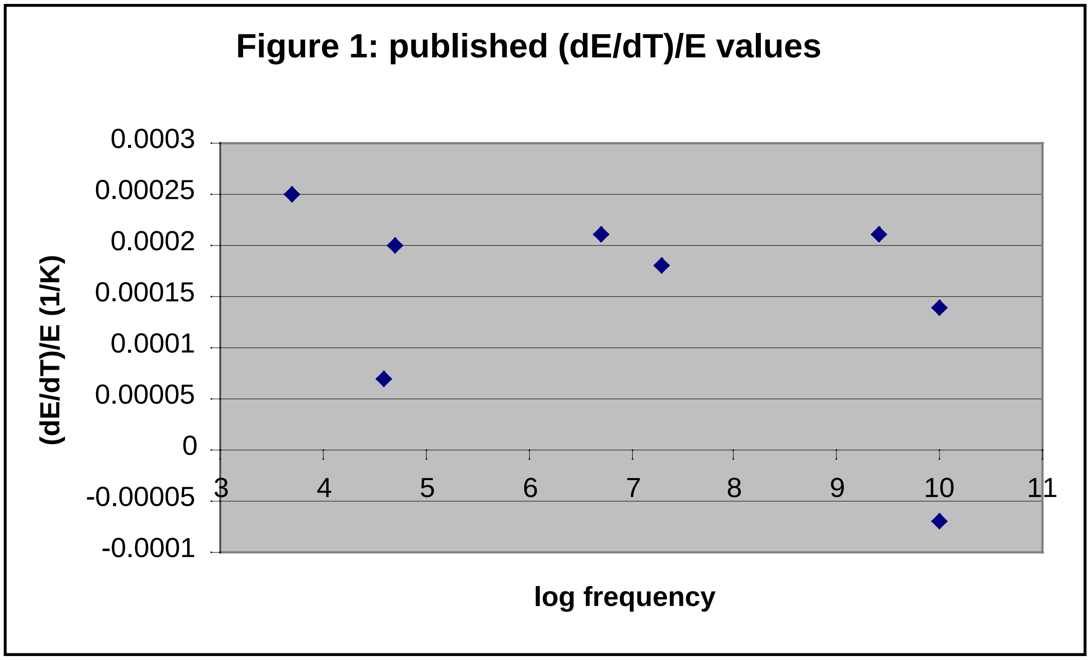
Nonlinear thermoelasticity, which arises when the Young’s modulus of a material depends on temperature, has recently been identified as potentially the largest source of suspension thermal noise for fused silica suspensions in LIGO II[1]. It can be easily seen how the thermal expansion responsible for linear thermoelastic damping can cause thermal fluctuations in the position of a mass suspended from a thin elastic wire. If we consider the wire as having a front half and a back half, it is obvious that a thermal fluctuation in the temperature of the front half of the wire will lead to an expansion or contraction of that part of the wire and therefore a bend in the wire, thus moving the mass. Now consider what happens if this wire is under fixed tension and the Young's modulus is a function of temperature. A thermal fluctuation in one half of the wire will change the Young's modulus, and therefore the strain of that segment under the tension. This also will bend the wire. It is easy to show that these two effects can exactly cancel each other if the tension is such that the static strain of the fiber is $u_0=\alpha E_0/E'$, where $\alpha$ is the linear expansion coefficient and $E_0$ and E' are the Young's modulus and its temperature derivative. Notice an important condition that must hold for this cancellation to occur: the sign of E’ must be positive, like that of $\alpha$, for the cancellation strain $u_0$ to be positive, which must be true for tension.

It has been shown that nonlinear thermoelastic damping varies as the square of the static strain $u_0$, and so the strain in the LIGO II suspensions must be carefully chosen to achieve this cancellation; too much strain leads to increased thermal noise, despite the dilution factor that also increases with static strain. Therefore the value of E’ needs to be known with some precision.

# 4 PRIOR WORK

Measurements of the temperature dependence of the Young’s modulus have been reported several times in the literature[2-9], with these measurements being made by different techniques, in different temperature ranges, and at different frequencies. Not surprisingly, the published values differ from each other. These results are displayed in Figure 1, which shows $E'/E_0$ vs. frequency (excepting the last two points for which no frequency data were given). The spread in these values is a factor of three, excepting an uncredited curve in the GE data sheets for “synthetic fused silica,” which shows $E'/E_0$ to be negative. We will abstain from any critical evaluation of these prior results, and simply take them at face value. The spread may then be interpreted as due to different samples, or different frequencies, or both. It is clear, then, that we at LIGO will need a measurement of E’ for our type of fused silica, in our frequency range. We now describe such a measurement.

**Figure 1: Published values of E'/E. The two data points at $10^{10}$Hz were published without the measurement frequency specified.**




# 5 EXPERIMENT

We measured the frequencies of cantilever bending modes of fibers drawn from Heraeus Suprasil fused silica on our glassworking lathe using a hydrogen/oxygen flame. The frequencies of the bending modes of a cantilever of length L clamped at one end are given by the solutions to the equation cos(kL)cosh(kL)+1=0, where k is given by

$$k^4=\omega^2\rho S/E_0 I,$$

with density ρ, cross-sectional area S, and moment of inertia I. This equation is satisfied by a set of values $C_i$, where $kL=C_i$, which we can rearrange in terms of ω as $\omega \propto E^{1/2}$. Temperature dependence of E then relates to that of ω by

$$E'/E_0=2\omega'/\omega_0=2f'/f_0,$$

independent of $C_i$.

This analysis is an approximation since L, S, I, and even ρ will also vary with temperature through the thermal expansion coefficient α. It is easy to show that for fused silica, α is so small that this effect can be ignored, and the approximation is good. It should be noted that the bending mode frequencies measured for our fiber are only crudely approximated by the equation above, probably due to the taper end of the fiber where it was drawn from the rod. This also does not affect the equation for $E'/E_0$.

We monitored the bending modes of the fiber by focussing a HeNe laser beam directly onto the fiber and collecting the forward-scattered light on a split photodiode. Although the forward scattering from the fiber is quite complicated, the difference in voltages of the two halves of the photodiode is linear with fiber displacement over a much larger range of fiber motion than that used in frequency measurements, as we determined by mounting the fiber on a translation stage and scanning it through the laser beam.

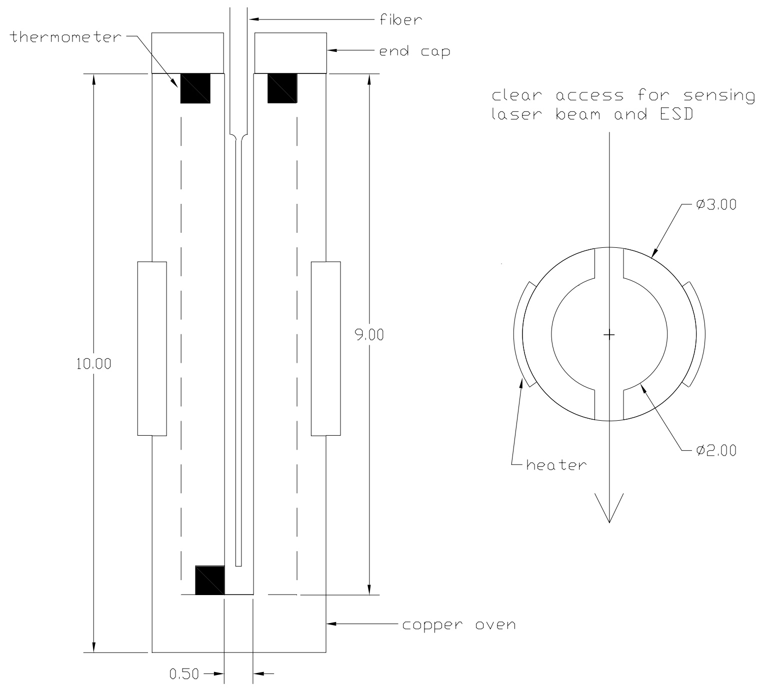
The experiment was performed in vacuum to reduce the effect of viscous damping by air friction on the fiber frequency. A viscous damping mechanism, such as air friction, will alter the mode frequency slightly, and fluctuations in the level of this damping could mask the frequency shift due to temperature change of E. No frequency measurement was trusted if the quality factor of the mode was less than $10^5$ for this reason.

The bending modes of the fiber were excited by an electrostatic drive (ESD), which consisted of a copper wire held about 1mm from the fiber and driven at the mode frequency with high voltage.

To measure the mode frequency precisely, the electrostatic drive was driven near the mode frequency until the fiber oscillated with an amplitude clearly visible above the noise level caused by electronic noise and acoustically excited fiber motion. The ESD was then shut off, and the photodiode signal during the mode ringdown was input to a lock-in amplifier with a stable reference oscillator about .5Hz away used as a reference. The lock-in output, a ~.5Hz sinusoid of decaying amplitude, was stored on a computer and analyzed later for both ringdown time (to verify high mode Q) and frequency, which was then added to the stable reference frequency to get the fiber mode frequency to within 2mHz, the precision set by the measurement time of 500s. This uncertainty is much less than the uncertainty of the temperature, as will be seen later.

The temperature of the fiber was varied by enclosing it in an oven, which is depicted in Figure 2. This oven was a hollow copper cylinder slightly longer than the fiber, with lengthwise slits on either side for passage of the laser beam and entry of the ESD. The cylinder was machined to a fair polish and so had a low emissivity. The inside of the cylinder had a layer of thin black plastic (a trash bag) glued to it to increase the emissivity to about .9. Two strip heaters were glued to either side of the oven to heat it, and thermometers at the top and bottom of the oven monitored the oven temperature. A copper cap with a slot to pass the fiber's rod end was pressed onto the top of the oven with an indium seal for thermal conduction. This cap was polished to a shine on the outside and oxidized black on the inside for high emissivity. The entire oven was also wrapped in aluminum foil for insulation and to 'seal' most of the slots, except for gaps for the laser and ESD.

**Figure 2: The oven used in the experiment.**




The ringdown data collected at first often showed nonexponential ringdown and low Q, were not very reproducible, did not fit a linear f/T curve well and showed different values of $E'/E_0$ for the different modes by ~20%. We then opened the vacuum and removed the fiber from the oven, and found that the plastic or glue in the hot oven had deposited some gunk on the fiber. We cleaned the fiber, reinstalled it in the oven, and collected more data, this time taking care not to exceed 60 °C in the oven. In addition, the different temperature data were collected in a randomized order, with two measurements at different times for each temperature, in order to check reproducibility. This time the reproducibility was quite good, the Q's of the modes were high and their ringdowns exponential, the f/T curves were quite linear, and the $E'/E_0$ values for the four modes agreed to within 3%.

The calibration of the fiber temperature required some calculation. Because the fiber was radiantly heated by the oven, and this oven had some gaps, the fiber was also radiatively cooled through these gaps to a lower temperature than that of the oven. Conductive cooling of the fiber through its clamp was insignificant due to the low thermal conductivity of fused silica and the fiber's high surface to volume ratio. The whole fiber was within the oven, with only the larger rod end sticking out at the top (although the rod does not penetrate far into the oven, motivating the addition of the oven cap).

The inside of the oven is assumed to have an emissivity of .9, as is the fiber, based on values reported in the CRC Handbook. The outside world has an emissivity of 1 (radiation exiting the oven is almost certainly absorbed before it is reflected back into the oven) and a temperature of 300K. The oven temperature is the average of the thermometer temperatures. The fiber is assumed to be uniformly illuminated by both the oven and the outside world through the gaps in the oven, which occupy 2.7% of the solid angle when the Al foil wrap is included. The fiber temperature is then found by using the Stefan-Boltzmann equation and assuming radiative balance between the fiber and its surroundings, and is given by the following equation.

$$T_{\text{fiber}} = 0.973 T_{\text{heater}} \left( \frac{27 \times (300K)^4}{4 \times 973 (T_{\text{heater}})^4} + 1 \right)$$

Clearly, the fiber temperature differs increasingly from the oven temperature the hotter the oven is. The error in the fiber temperature is estimated as this difference between the estimated fiber and oven temperatures (thus estimating the calculation as being uncertain to a factor unity) combined with the uncertainty of the thermometers, taken from data sheets. The total uncertainty in fiber temperature ranges from .5 C at 22 C to 1.6 C at 53 C, and is the dominant source of uncertainty in this measurement, making $E'/E_0$ accurate to about 5%.

# 6 RESULTS AND DISCUSSION

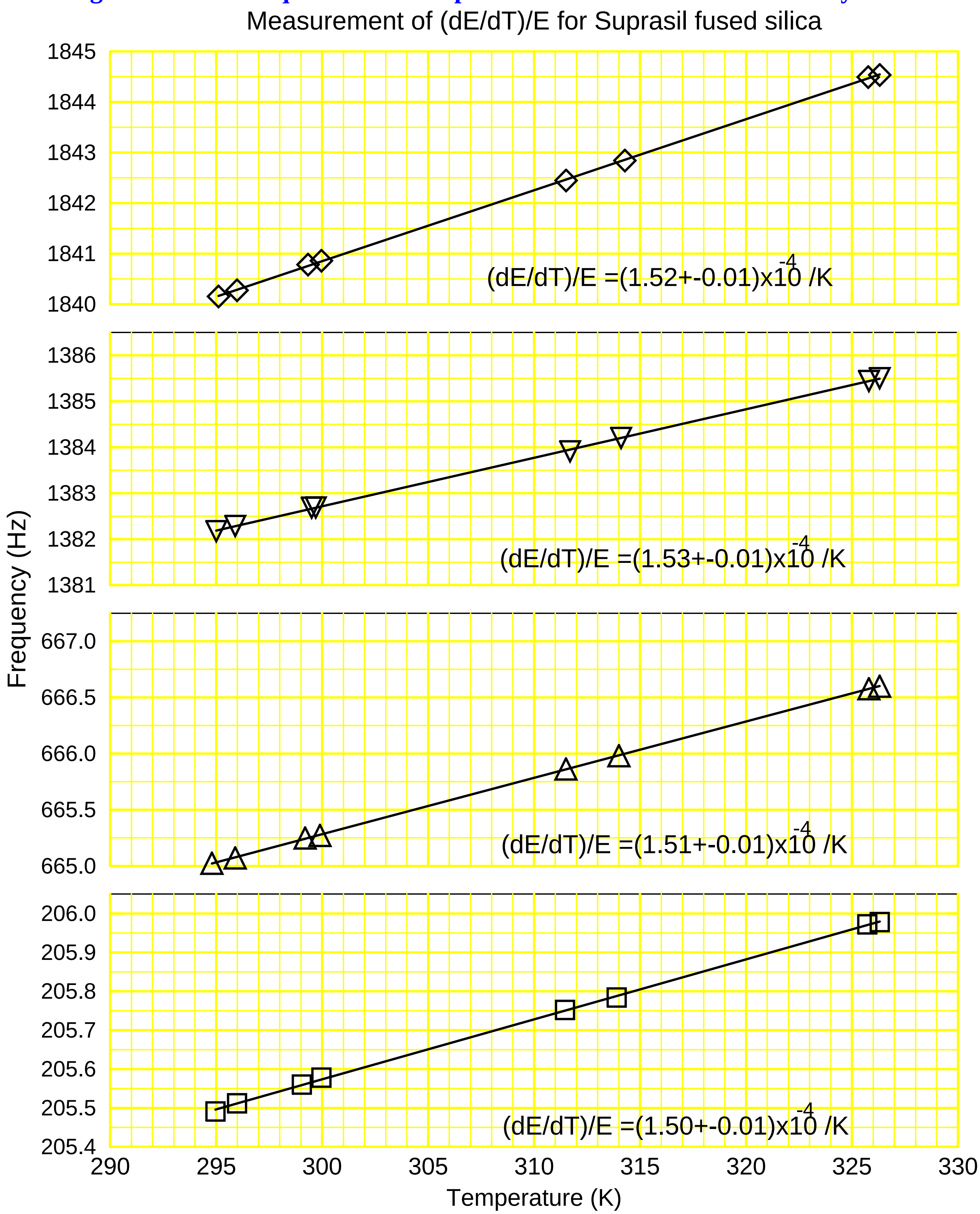
The data are shown in Figure 3. The values for the various modes are, as mentioned, in agreement at an average value of E'/E =+1.52e-4/K± 5%, which is within the range of previously reported values. We first note that, like nearly all the other measurements, E'/E is positive. Therefore a suitably chosen tension will cancel the thermoelastic and nonlinear thermoelastic noise. Based on the known values for E and $\alpha$, the optimum stress that achieves this cancellation is 240MPa.

This measurement should be the one used for LIGO II calculations for the following reasons. First, this measurement uses the same type of fused silica LIGO will likely use in its suspensions. Second, this measurement includes data at the temperature likely for LIGO II, and at 200Hz, a lower frequency than any other measurement and the one closest to the important frequency range for suspension thermal noise. Finally, this measurement analyzes bending modes, which are the type of motion responsible for thermal noise in LIGO II suspensions. Other measurements employ Brillouin scattering or ultrasonic longitudinal modes to measure E'/E. While we do not claim that there is anything wrong with these other types of measurement, given the discrepancies between the reported results, in the absence of a clear explanation for the differences it is prudent to trust the data collected as closely to the parameters of interest as possible.

# 7 WHAT MORE SHOULD BE DONE

The suspension thermal noise is important down to 10 Hz, so data to this frequency would be useful. This data was difficult to collect in our apparatus due to seismically excited noise motion of the fiber. It would also be valuable to repeat this measurement for other samples of fused silica. Ultimately, the nonlinear thermoelastic damping itself should be measured and verified, rather than just the parameters that determine it.

**Figure 3: Mode frequencies vs. temperature and the values of E’/E they show.**



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