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Use of Magnets in the Suspension Design

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1 ABSTRACT

The suspension will use small permanent magnets to provide damping and attitude control of the optics. This document summarizes a number of calculations which were done to ensure that the use of magnets was thoroughly understood. These include

- comparison of the measured magnet strength with the spec-sheet value,
- a first-principles calculation of eddy current damping,
- a check on the method of calculation by comparison with the experiment results of Miyoki (LIGO-T970073-00-D),
- eddy current damping calculations for the optic magnets to the sensor/actuator brackets, for the optic magnets to the static-dissipation plating on the sensor/actuator heads, for the optic magnets to the pitch-adjustment magnets, and for the pitch-adjustment magnets to the wire loop supporting the optic,
- and an upper limit on the conductivity of the plating.

2 KEYWORDS

LOS, SOS, magnet, PAM, OSEM, sensor/actuator

3 REVISION HISTORY

11/1/00 - Pre-Rev 00 Draft

11/9/00 - Rev 00 release. No significant changes from draft of 11/1/00

4 REFERENCES

LIGO-T950011-19-D, S. Kawamura and F. Raab, Suspension Design Requirements

LIGO-T960073-00-D, S. Miyoki, S. Kawamura, M. Fine, J. Hazel, and F. Raab, Loss due to Eddy Current Damping between Magnets and Sensor/Actuator Head Holders in the Small Optics Suspension

5 OVERVIEW

Both the LOS and SOS designs will use magnets bonded to the optics together with magnetic coils in the sensor actuator heads to provide damping and attitude control for the optics. In addition, the LOS design will use pitch-adjustment magnets (PAMs) opposing those on the optics to provide extra static torque to bridge the gap between the precision attainable by manual adjustment and the dynamic range of the coil actuators.

A potential problem with this design is the eddy-current damping between the magnets and various metal objects both on the optic and in the surrounding space. Miyoki measured the eddy current damping between a magnet and an aluminium duplicate of the SOS sensor/actuator head holder. (The damping in Al is 30 times greater than for stainless steel because of the lower resistivity and thus easier to measure.) He did not attempt to estimate the damping analytically. However there are many other combinations of magnet and conductor to be checked, and so the remainder have been calculated, using Miyoki's result as a check on the method of calculation.

The third potential problem is cross-coupling between the pitch, yaw and axial degrees of freedom due to forces from the pitch adjustment magnets, especially in the presence of strength and position mismatches in the PAMs. These are calculated and shown to be negligible.

6 ESTIMATION OF THE MAGNET STRENGTH

6.1. Method 1: Using data in spec sheet

The magnets chosen to be bonded to the optic in both LOS and SOS are l =3.175 mm long and a =0.9525 mm in radius, made of NEO-35. The dipole moment of a volume V of uniform magnetization M is

$$p_m = MV = \pi M l a^2$$

The magnetization can be estimated in terms of the residual induction, which is about 1.25 T for NEO-35:

$$M \approx \frac{B_{resid}}{\mu_0}$$

In fact, B_{resid} is the value of B at a point on the hysteresis curve where H has just been reduced to zero, whereas the H field inside a bar magnet is substantial. A calculation using formulae in M.A. Plonus, "Applied Electromagnetics", shows that the average axial $B/(\mu_0 H)$ inside the magnet is -3.6, which, by inspection of the hysteresis curve for NEO35, is still (just) on the flat, top part of the curve, so the equilibrium B field should still be quite close to B_{resid}. Thus, p_m =0.0090 J/T. This is an upper bound - small regions of high negative H near the pole faces may result in partial local demagnetization.

6.2. Method 2: Pick-up test

In this method, one magnet was used to pick up another, and the distance between them at the point the gravitational force was overcome was used together with the mass to calculate the dipole moment using the formula

$$mg = F_z = \frac{3\mu_0 p_m^2}{2\pi z^4}$$

The value from this method was $p_m = 0.0079$ J/T.

6.3. Method 3: Tube test

In this test two magnets were confined in a narrow vertical glass tube so as to align them and the distance at which the upper one floated above the lower one was measured. The formula of the previous section was used to derive $p_m = 0.0073$ J/T.

6.4. Preferred value

As expected the measured values were somewhat lower than the value derived (naively) from the spec sheet. The tube and pick-up tests give similar results. The tube result ($p_m = 0.0073$ J/T) is used in subsequent analysis.

7 EDDY CURRENT DAMPING

7.1. Requirements

Eddy current damping is velocity dependent, so that the lag angle ϕ at any particular frequency, (on- or off- resonance) is given by

$$\phi(\omega) = \frac{\omega\gamma}{\omega_0^2} = \frac{\omega b}{\omega_0^2 m}$$

where b is the damping force per unit velocity.

In T950011, Suspension Design Requirements, the maximum acceptable lag angles for eddy current damping are given as

$$\phi_{MAX-LOS}(f) = 7.5 \times 10^{-7}\left(\frac{f}{100\mathrm{Hz}}\right)$$

and

$$\phi_{MAX-SOS}(f) = 2 \times 10^{-5}\left(\frac{f}{100\mathrm{Hz}}\right)$$

With masses of 10.2 and 0.25 kg, and frequencies of 0.744 and 1.0 Hz, these imply damping factors of

$$b_{MAX-LOS}(f) = 0.36 \times 10^{-6}\mathrm{N}/(\mathrm{m}/\mathrm{s})$$

and

$$b_{MAX-SOS}(f) = 0.31 \times 10^{-6}\mathrm{N}/(\mathrm{m}/\mathrm{s})$$

7.2. Theory

The general case for eddy current damping is very difficult to calculate. Fortunately the suspension can be approximated by a series of situations with axial symmetry which are more tractable.

Consider a dipole fixed at the origin and oriented along the z axis. Then the axial and radial components of the magnetic field in cylindrical coordinates are

$$B_z = \frac{\mu_0 m_z}{4\pi}\frac{r^2 + 2z^2}{(r^2 + z^2)^{5/2}}$$

$$B_r = \frac{3\mu_0 m_z}{4\pi}\frac{rz}{(r^2 + z^2)^{5/2}}$$

Now consider a block of metal axially symmetric about the z axis and moving in the z direction with velocity v_z. Because of the axial symmetry there is no tendency for current to flow radially. Thus we can divide the block into many toroidal annuli of infinitesimal cross-sectional area dS and consider the current in each one independently. The EMF in each annulus can be calculated either as the flux of B_r cut per unit time or as the negative rate of change of the B_z integrated over the area of the loop:

$$e = \oint E \cdot dl = \int B_r \times v_z \cdot dl = -\frac{\partial}{\partial t}\int B_z \cdot dS$$

Since the annulus is circular, B_r is constant in magnitude over the perimeter and it is convenient to use the flux-cutting formula, so that the EMF is

$$e = 2\pi r B_r v_z$$

If the resistivity of the metal is ρ, then the conductance of the loop is

$$dG = \frac{dS}{2\pi r\rho} = \frac{dr dz}{2\pi r\rho}$$

The power dissipated in the annulus is then

$$dP = e^2 dG$$

and the contribution to damping force per unit velocity is

$$db = \frac{dP}{v_z^2} = \frac{9\mu_0^2 m_z^2}{8\pi\rho} \frac{r^3 z^2 dr dz}{(r^2 + z^2)^5}$$

7.3. Analysis of Miyoki's experiment

7.3.1. Estimation of the magnetic field and EMF

The sensor/actuator head holder used by Miyoki consists of a bar 395 mm long, 65 mm tall and 16 mm thick, with a 27 mm diameter hole slightly below the midline about 1/3 of the way along. The magnet was placed on and parallel to the axis of the hole. Although the geometry of distant parts of the holder is complicated, the strength of the interaction falls very strongly with distance and so only the axially symmetric part around the hole contributes significantly. Integrating from $z = 10.59$ mm (the separation of 9 mm plus half the magnet length) to $z = 26.59$ mm (the same plus the holder thickness) and from $r = 13.5$ mm (the hole radius) to $r = 28.5$ mm (the largest circle centred on the hole and fully contained in the holder metal) and using $\rho_{Al} = 2.65\times10^{-8}\ \Omega$ m gives $b = 4.5\times10^{-6}$ N/(m/s).

7.3.2. Comparison with Miyoki's value

Miyoki specifies a value for the damping at resonance $\phi_0 = \phi(\omega_0)$ of the test pendulum of $\phi_{meas} = 5.3\times10^{-5}$ (or, restoring a significant figure, 5.27×10^{-5}). Given m=5.7 g and $\omega_0/(2\pi) = 0.9$ Hz, $b_{meas} = 1.7\times10^{-6}$ N/(m/s). This is 2.6 times smaller than estimated above. The reason for the discrepancy has not been identified yet, but it is likely to be small errors amplified by the high powers of r that appear in the damping formula.

7.3.3. Derivation of damping factor for SOS

Digression: from the above, Miyoki estimates the damping and thermal noise in the real system with four magnets instead of one, a frequency of 1.0 Hz instead of 0.9, and a stainless steel holder of 30 times lower conductivity. He does not give the details of this calculation but it can be reconstructed as follows. The damping force under these conditions which implies a damping factor of

$$\phi_{0(SOS)} = \frac{8\pi b_{meas}}{\omega_0 m}\frac{\rho_{Al}}{\rho_{SS}} = 1.44\times10^{-7}$$

However Miyoki's quoted value is 1.8×10^{-7}, which is different by a factor of 0.9^2, apparently due to a mistake. Miyoki's values for thermal noise are consistent with his quoted value, so the real situation is approximately 20% less stringent than he calculates.

7.4. Eddy Current Calculations for the real system

7.4.1. Optic Magnets to Sensor/Actuator Brackets

We take a value based on Miyoki's measurement as being slightly more conservative than the calculated value.

$$b_{HOLDER} = 4b_{meas}\frac{\rho_{Al}}{\rho_{SS}} = 2.26\times10^{-7} \qquad \mathrm{N/(m/s)}$$

This is narrowly within the specification for both LOS and SOS.

7.4.2. Optic Magnets to Sensor/Actuator Plating

The damping due to currents induced in the conductive plating on the sensor/actuator body is mostly due to the plating on the end face. It can be conservatively approximated as an annulus with outer radius equal to the radius of the body, and inner radius equal to the smallest dimension of the rectangular hole in the end. The damping is inversely proportional to the resistivity. The reference resistivity is that measured for a prototype sensor/actuator with gold plating. This gives a damping factor of

$$b_{END} = 4\times1.38\times10^{-9}\cdot\left(\frac{2.67\Omega/\mathrm{SQ}}{\rho}\right) \qquad \mathrm{N/(m/s)}$$

The damping due to the cylindrical surface is much less:

$$b_{CYL} = 4\times3.14\times10^{-11}\cdot\left(\frac{2.67\Omega/\mathrm{SQ}}{\rho}\right) \qquad \mathrm{N/(m/s)}$$

The damping for the end face is close to being problematic. The reference resistivity gives about a factor of 50 in hand. Therefore we set a lower limit of 0.25 Ω/SQ, keeping a factor of about 5 for calculation errors. (The upper limit as far as dissipating charge is concerned is very large. 1 $\mathrm{M}\Omega/SQ$ is more than adequate. However if the plating material is a highly conductive one such as gold, this implies a ridiculously thin layer which is likely to be broken up by the friction of putting the sensor/actuator into the holder. Therefore for metallic plating we specify a very loose upper limit of 25 Ω/SQ.)

7.4.3. PAM Magnets to Suspension Wire

The damping factor due to currents induced in an suspension wire due to motion relative to the PAM magnet is trivial. For example, for LOS:

$$b_{LOSWIRE} = 4 \times 8.10 \times 10^{-14} \qquad \mathrm{N/(m/s)}$$

7.4.4. PAM Magnets to and from Optic Magnets

The damping factor due to currents induced in the optic and PAM magnets due to their relative-motion is trivial:

$$b_{MM} = 2 \times 4 \times 1.87 \times 10^{-11} \qquad \mathrm{N/(m/s)}$$

7.4.5. PAM Magnets to Magnet Standoffs

The damping factor due to currents induced in one of the dumbell standoffs due to motion relative to the PAM magnet is just a bit bigger due to the higher conductivity of aluminium, but still trivial:

$$b_{MS} = 4 \times 9.41 \times 10^{-11} \qquad \mathrm{N/(m/s)}$$

This is a conservative estimate which ignores the narrowing at the waist of the dumbell.

7.4.6. Optic Magnets to PAM Screws

The damping factor due to currents induced in the PAM screw due to motion relative to the optic magnet is trivial:

$$b_{MS} = 4 \times 5.51 \times 10^{-11} \qquad \mathrm{N/(m/s)}$$