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A possible thermal noise measurment using the 40-meter

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ABSTRACT

Is it worthwhile to attempt a thermal noise measurement at the 40-meter? In this document I will try to answer, at least partially, that question by looking at the level of thermal noise we may expect from such an attempt and how close it will come to that of LIGO's.

KEYWORDS

Thermal noise; Sapphire; LIGO II; 40 meter; TNI

1 INTRODUCTION

Braginsky has predicted two new internal mirror thermal noise sources that may affect our choice of mirror material in, and ultimately the sensitivity of, LIGO II [1]. It behooves us to study these noise sources experimentally. We have been building a dedicated interferometer to study thermal noise, the Thermal Noise Interferometer (TNI), but this instrument is limited by small spot size and may not be well suited for studying these new noise sources. This was not expected to be much of a problem when the only noise source we knew about was the Brownian motion associated with structural damping, but Braginsky's new noise predictions make spot size more of an issue than was previously expected.

One way to increase spot size in a Fabry-Perot cavity is to increase its length, and it has been proposed [2] that the 40-meter be converted to make a thermal noise measurement. In this document, I will propose a geometry for a 40-meter cavity that could be used to measure thermal noise, and I will look at its spot size and noise performance compared to both the TNI and a full-scale LIGO.

2 Spot size and cavity length

It is useful to look at how spot size depends on cavity length and radius of curvature of the mirrors. For simplicity, let's assume our cavities are symmetric. Then the spot size on each mirror is [3]

$$\omega = \sqrt{\frac{L\lambda}{\pi}\sqrt{\frac{1}{1-g^2}}},$$

where L is the length of the cavity, λ is the wavelength of light used, and $g = 1 - L/R$ is the geometry factor of the cavity, where R is the radius of curvature of the identical end mirrors.

If we preserve the relative geometry of the cavity by keeping g constant, then we see that the spot size only grows with the square root of the cavity length, or

$$\omega \propto L^{\frac{1}{2}}.$$

On the other hand, if we keep the cavity length fixed and vary the radius of curvature of the mirrors, then the spot size grows more slowly. For a nearly-unstable resonator, with $R/L \ll 1$,

$$\omega \propto R^{\frac{1}{4}}.$$

If you want to make an interferometer with a large spot size, then, you get more out of making it longer than you do from making the mirrors flatter. Moreover, because the spot size grows so slowly with both length and radius of curvature, you don't need a very big interferometer to get close to the spot size in LIGO's 4 km arm cavities.

Below is a table that lists spot sizes for the Thermal Noise Interferometer, LIGO, and a proposed 40-meter interferometer that uses standard LIGO ITM's as its mirrors. These ITM's have a radius of curvature of $14.2km$, which gives a spot in a $40m$ cavity comparable to that of LIGO's $4km$ cavities. Thermal lensing and fabrication should not be a problem, provided they are not a problem for LIGO, but alignment may be difficult.

IFO	L	R	g	ω
TNI	1.0 cm	1.0 m	0.99	155 μm
Proposed 40 m	40.0 m	14.2 km	$1 - 2.8 \times 10^{-3}$	1.3 cm
LIGO	4.0 km	$\cdots$	1/3	3.8 cm

3 Spot size and noise

The question naturally arises, how will the noise performance of this proposed 40-meter differ from the TNI's and from LIGO's? It is instructive to look at scaling relations for three types of thermal noise, Brownian noise, thermoelastic noise, and photothermal noise. Using Braginsky's formulae [1], Brownian noise scales as $\omega^{-1/2}$. Using the spot sizes listed in the table above, the thermal noise interferometer's Brownian noise will be about 15.7 times that of LIGO's. (All of these comparisons are between levels of *displacement* noise, with units of length/$\sqrt{Hz}$.)

$$\frac{S_{B,TNI}^{1/2}}{S_{B,LIGO}^{1/2}} = \left(\frac{\omega_{TNI}}{\omega_{LIGO}}\right)^{-\frac{1}{2}} = 15.7$$

The Brownian noise in our proposed 40-meter would be about a factor of ten below that, nearly down to what LIGO's is expected to be.

$$\frac{S_{B,40m}^{1/2}}{S_{B,LIGO}^{1/2}} = \left(\frac{\omega_{40m}}{\omega_{LIGO}}\right)^{-\frac{1}{2}} = 1.7$$

Thermoelastic noise scales relatively quickly with spot size, and the thermal noise interferometer's small spot size rules out any realistic study of this noise source on the level LIGO is concerned with.

$$\frac{S_{TE,TNI}^{1/2}}{S_{TE,LIGO}^{1/2}} = \left(\frac{\omega_{TNI}}{\omega_{LIGO}}\right)^{-\frac{3}{2}} = 3.8 \times 10^{3}$$

Our proposed 40-meter interferometer would do better in this area.

$$\frac{S_{TE,40m}^{1/2}}{S_{TE,LIGO}^{1/2}} = \left(\frac{\omega_{40m}}{\omega_{LIGO}}\right)^{-\frac{3}{2}} = 4.63$$

Photo-thermal noise scales the fastest of the three with spot size, varying as ω^{-2}. This noise source will also be vastly larger in the TNI than in LIGO.

$$\frac{S_{PT,TNI}^{1/2}}{S_{PT,LIGO}^{1/2}} = \left(\frac{\omega_{TNI}}{\omega_{LIGO}}\right)^{-2} = 7.0 \times 10^{4}$$

In our proposed 40-meter, photo-thermal noise should be less than a factor of ten above LIGO's level.

$$\frac{S_{PT,40m}^{1/2}}{S_{PT,LIGO}^{1/2}} = \left(\frac{\omega_{40m}}{\omega_{LIGO}}\right)^{-2} = 7.7$$

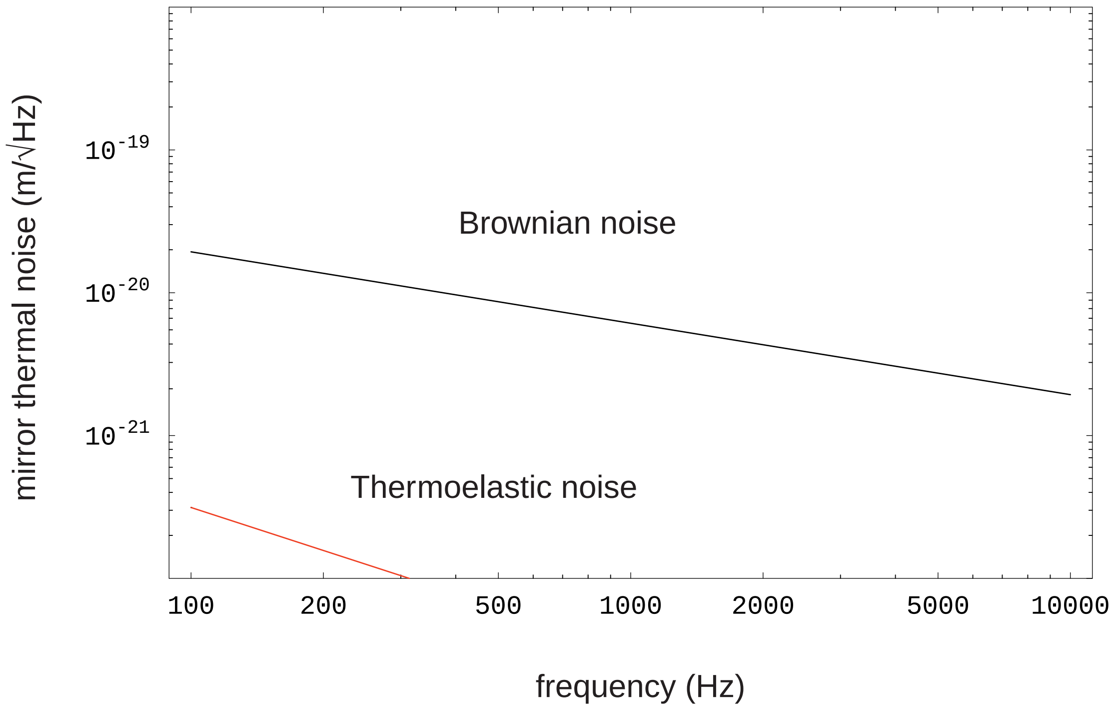

Figure 1: Expected noise in LIGO with fused silica mirrors. Brownian motion dominates.

4 Noise estimates

Here are some estimates of the relative sizes of the Brownian noise and thermoelastic noise in LIGO, the Thermal Noise Interferometer, and our proposed 40-meter with both fused silica and sapphire mirrors. I have not plotted photothermal noise here because it is expected to be both small and strongly dependent on the surface conditions of the mirrors. For each plot I have used Braginsky's formulae and parameter values [1].

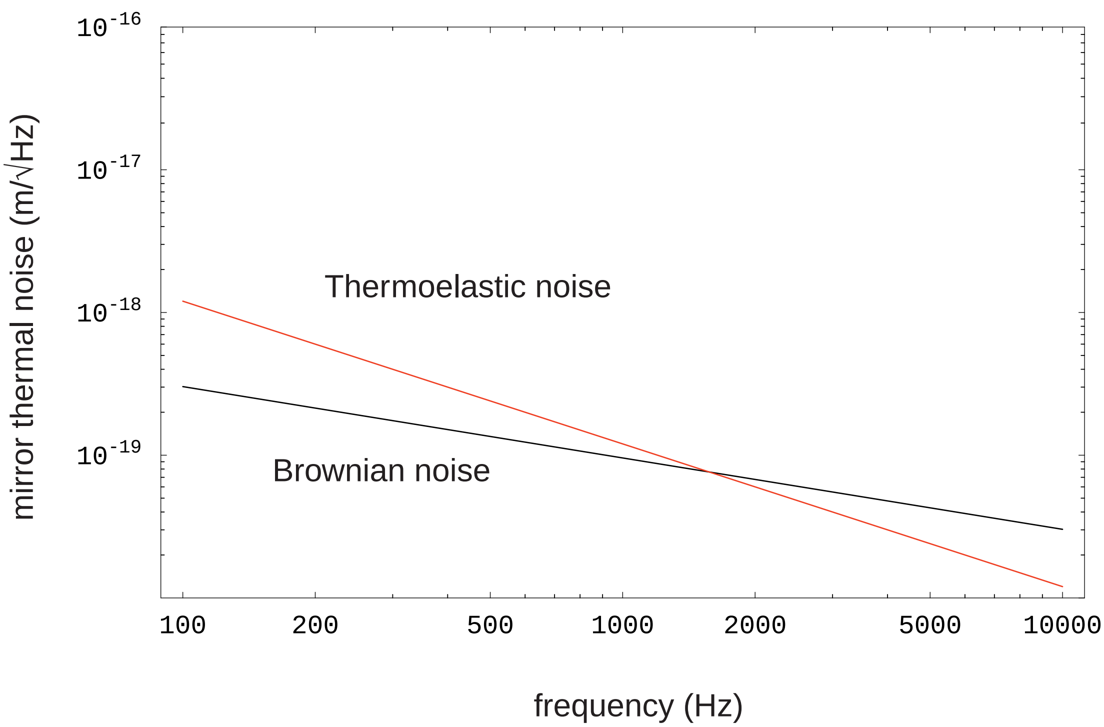

Figure 2: Expected noise in the TNI with fused silica mirrors. Note the change in scale relative to Figure 4. The crossover between noise sources around one kilo-Hertz would be very interesting to observe.

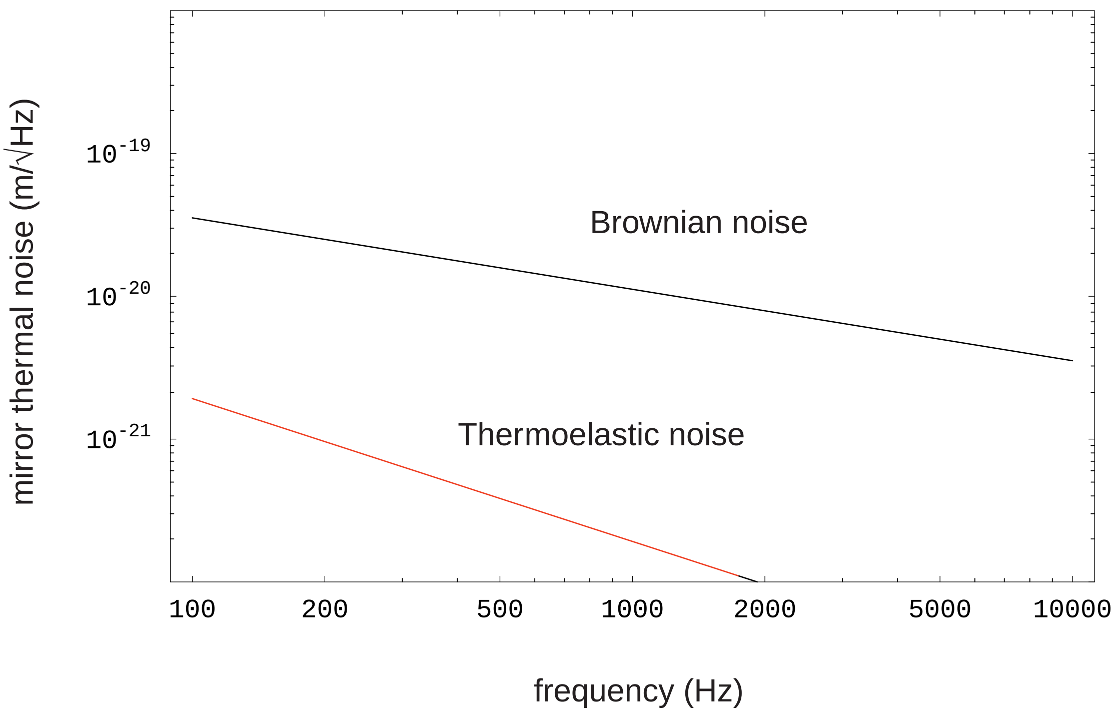

Figure 3: Expected noise in the proposed 40-meter with fused silica mirrors. This should closely mimic LIGO's thermal noise curve. Note that this scale is the same as that of Figure 4.

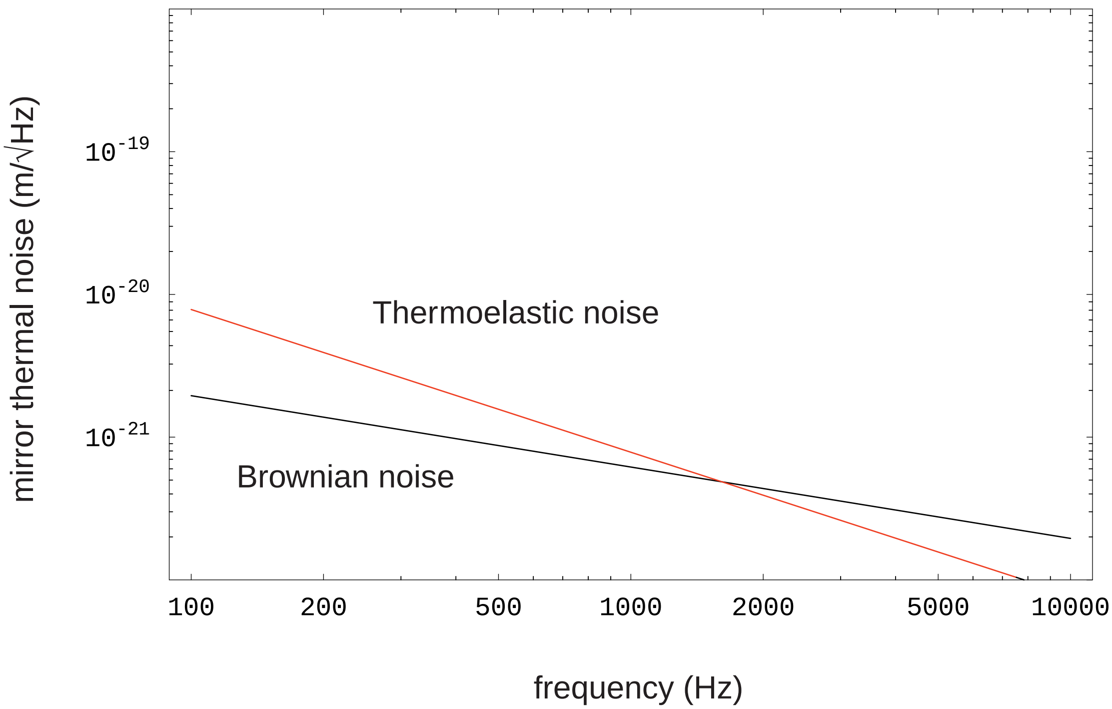

Figure 4: Expected noise in LIGO with sapphire mirrors and unchanged spot size. Thermoelastic noise is not nearly so much of a problem as in Braginsky's plots, because he underestimated our spot size.

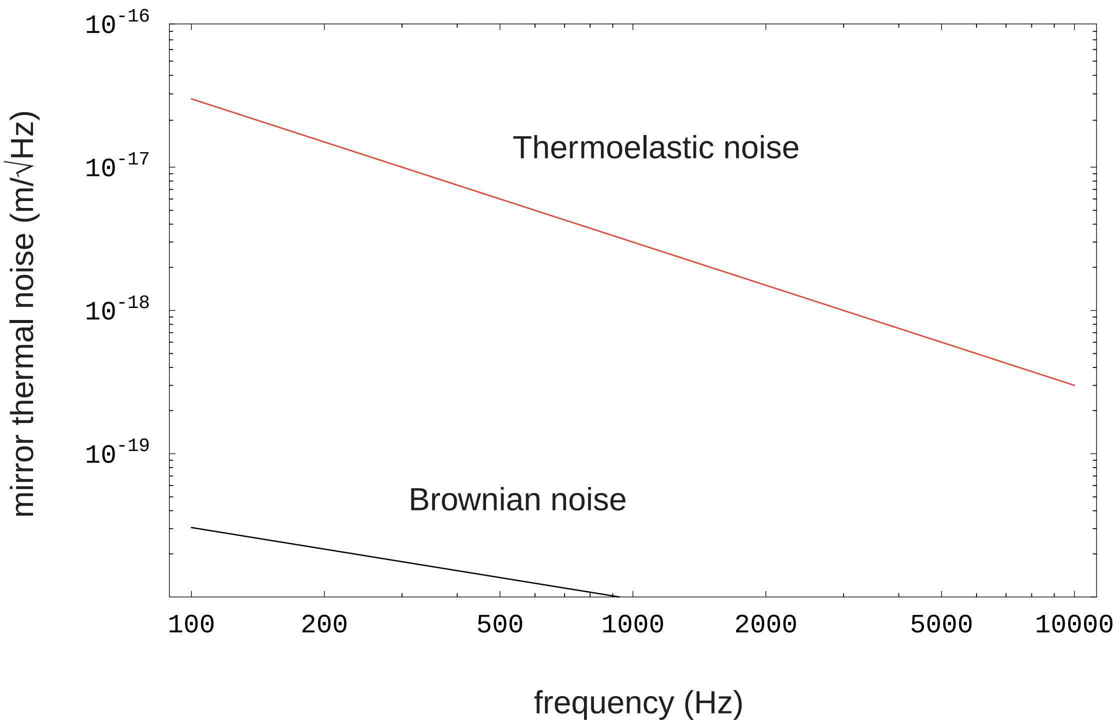

Figure 5: Expected noise in the TNI with sapphire mirrors. Thermoelastic noise is huge. This should provide an initial confirmation of Braginsky's thermoelastic noise prediction.

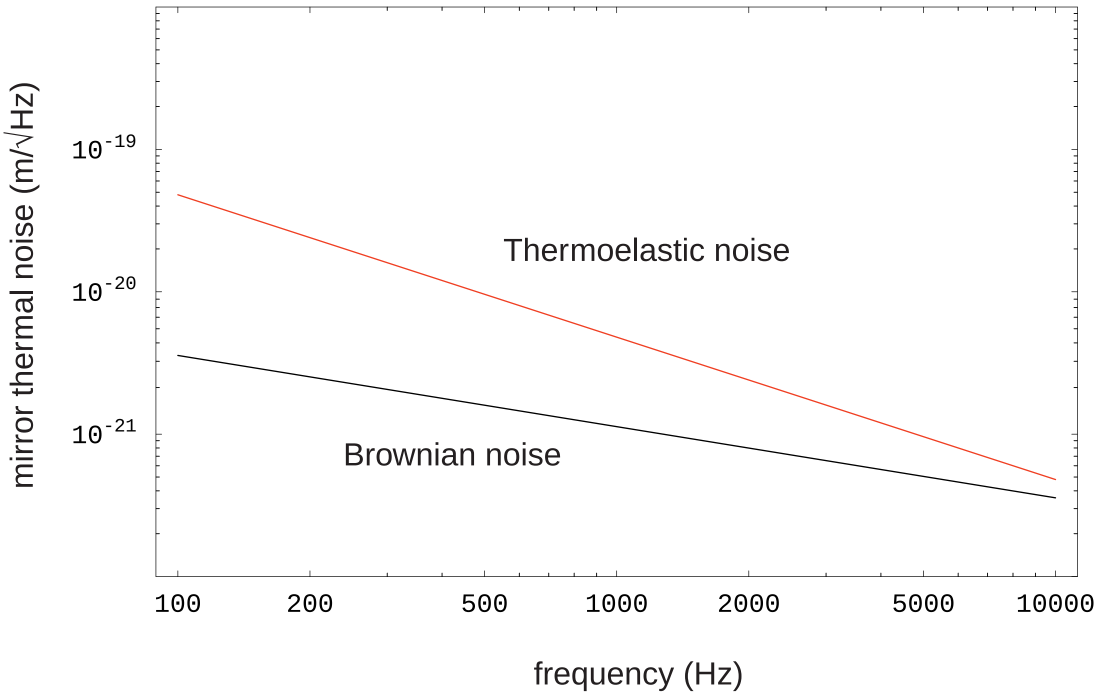

Figure 6: Expected noise in the proposed 40-meter with sapphire mirrors. Thermoelastic noise still dominates, but it is closer to LIGO's expected value.

That thermoelastic noise can dominate in a 40-meter interferometer leads one to ask, has it already been seen in the old 40-meter's configuration? It seems unlikely. A quick calculation shows that, with a g-factor of $1/3$, Brownian noise still dominates in a 40-meter interferometer.

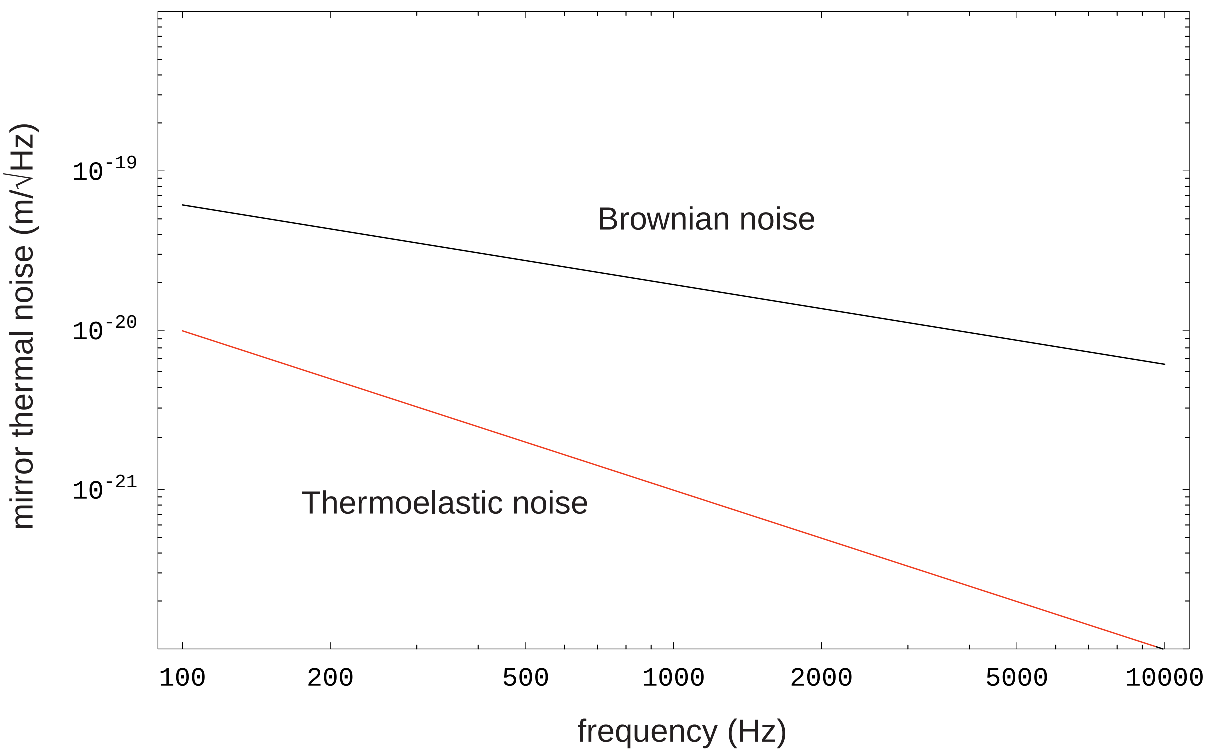

Figure 7: Could the old 40-meter have seen thermoelastic noise? Not likely, if its g-factor were $1/3$ as is LIGO's. Brownian noise would still have been dominant.

5 Conclusions

We should attempt to measure thermal noise using the 40-meter. If we could measure internal mirror thermal noise using the proposed topology (40-meter cavities using standard LIGO ITM's for mirrors), then it should be very close to LIGO's for all three mechanisms treated here: Brownian noise, thermoelastic noise, and photothermal noise. This is a dramatic improvement over the expected signal from the TNI, which we expect will be limited by its small spot size.

A thermal noise measurement using this geometry, $L = 40m$ and $R = 14.2km$, might be difficult because the cavities would be nearly unstable. Alignment could be difficult, but the TNI can help with this by serving as a pathfinder experiment. It's geometry is nearly as close to unstable, with $L = 1cm$ and $R = 1m$. If we could achieve decent alignment in the TNI with this geometry, then we might be able to do so in the 40-meter as well.

The TNI should be used to get an initial measurement of Braginsky's noise sources in both fused silica and Sapphire and to study alignment of a nearly-unstable resonator. What we learn from the TNI can then be applied to the thermal noise measurement in the 40-meter. Once the 40-meter sees mirror thermal noise, the TNI will be free to study suspension thermal noise, using a superattenuator for seismic isolation. Both experiments should be pursued simultaneously.

References

[1] V. B. Braginsky, M. L. Gorodetsky, S. P. Vyatchanin, *Thermodynamic fluctuations and photothermal shot noise in gravitational wave antennae*, Phys. Lett. A **264**, 1–10 (1999).

[2] Riccardo DeSalvo, private communication.

[3] A. E. Siegman, *Lasers*, University Science Books, Sausaligo, California (1986).