Calculation of the Electric field at the surface of the coating versus the transmittance of the stack

The aim of this paper is to calculate the electric field at the surface of the coating.

T : transmittance of the stack (= 0.014 @ 1064 nm for the ITM mirror)

Equation (a) : (ρ is the amplitude reflection coefficient of the stack)

the reflectance ρ of the stack interface between an incident medium of admittance η_0 and the stack of admittance Y is given by :

$$\|\rho\| = \sqrt{1-T} \quad \text{and}$$

$$\|\rho\| = \frac{\|\eta_0 - Y\|}{\|\eta_0 + Y\|}$$

In our case η_0=1 (incident medium is vacuum)

So : equation (a) $$\frac{\|1-Y\|}{\|1+Y\|} = \sqrt{1-T}$$

Equation (b) : the continuity of the electric field at the surface interface :

$$\|E_t\| = \|E_i + E_r\| \quad \text{(Ei = 27.46 v/m for P=1 W/m}^2\text{)}$$

E_t : transmitted electric field ; E_i : incident electric field and E_r (=ρ*E_i): reflected electric field.

$$\|E_t\| = \|E_i + E_r\| = \|E_i + \rho E_i\| = \|E_i\| * \|1+\rho\| = \frac{2}{\|1+Y\|} * \|E_i\|$$

We obtain finally for equation (b) : $\|1+Y\| = \dfrac{2\|E_i\|}{\|E_t\|}$

And (a) gives $$\|1-Y\| = \frac{2\|E_i\|}{\|E_t\|} * \sqrt{1-T}$$

Let Y=a+i.b so we obtain :

$$\|1-a-ib\| = \frac{2\|E_i\|}{\|E_t\|} * \sqrt{1-T} \quad \text{and} \quad \|1+a+ib\| = \frac{2\|E_i\|}{\|E_t\|}$$

$$(1-a)^2 + b^2 = \left(\frac{2\|E_i\|}{\|E_t\|}\right)^2 * (1-T) \quad \text{and} \quad (1+a)^2 + b^2 = \left(\frac{2\|E_i\|}{\|E_t\|}\right)^2$$

Substracting the 2 last equations, we obtain : $$a = \frac{\|E_i\|^2}{\|E_t\|^2} * T \quad (1)$$

So we can now extract the imaginary part of Y ,

$$b^2 = \left(\frac{2\|E_i\|}{\|E_t\|}\right)^2 - (1+a)^2 = \left(\frac{2\|E_i\|}{\|E_t\|} + 1 + a\right) * \left(\frac{2\|E_i\|}{\|E_t\|} - (1+a)\right)$$

Or $b^2 > 0$, so $\frac{2\|E_i\|}{\|E_t\|} - (1+a) > 0$

Replacing a by (1), we obtain the relation : $\|E_t\|^2 - 2\|E_i\|\|E_t\| + \|E_i\|^2 T < 0$, which is the equation of a parabole. Resolving the 2 roots of this equation, we obtain a condition concerning E_t.

The 2 roots are : $\|E_t\| = \|E_i\|(1+\sqrt{1-T})$ and $\|E_t\| = \|E_i\|(1-\sqrt{1-T})$, so we obtain, whatever the stack is a condition on E_t :

$$\|E_i\|(1-\sqrt{1-T}) < \|E_t\| < \|E_i\|(1+\sqrt{1-T})$$

If we make numerical application, we obtain for the ITM, T =0.014 and Ei = 27.46 V/m :

$$0.193 V/m < \|E_t\| < 54.73 V/m$$

So, in conclusion, for the ITM mirror, it’s impossible to reach values below 0.193 V/m @ 1064 nm