

GRAVITATIONAL-WAVE ASTRONOMY WITH INSPIRAL SIGNALS OF SPINNING COMPACT-OBJECT BINARIES

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ABSTRACT

Inspirals signals from binary compact objects (black holes and neutron stars) are primary targets of the ongoing searches by a number of ground-based gravitational-wave interferometers (LIGO, Virgo, GEO-600 and TAMA-300). Detection of such inspirals and ensuing mergers is expected to provide us with important physical information about the properties of the sources, bearing on outstanding issues in compact-object astrophysics, including the progenitors of short γ -ray bursts. Compact-object spin effects add to the challenges associated with searches and anticipated detections, but on the other hand they provide some interesting possibilities for extracting astrophysical information. We present parameter-estimation simulations for inspirals of black-hole binaries with neutron-star companions using Markov-Chain Monte-Carlo methods. We specifically highlight the potential for measurements of masses, spins, source sky location and distance of such objects with just one or two gravitational-wave detectors.

Subject headings: Stars: Binaries: Close, Stars: White Dwarfs, Gravitational Waves

1. INTRODUCTION

Binary systems with compact objects – neutron stars (NS) and black holes (BH) – in the mass range $\sim 1M_{\odot} - 100M_{\odot}$ are among the most likely sources of gravitational waves (GWs) for ground-based laser interferometers currently in operation (Cutler & Thorne, 2002): LIGO (Barish & Weiss 1999) Virgo (Arcese et al. 2004), GEO-600 (Willke et al. 2004) and TAMA-300 (Takahashi et al. 2004). The three LIGO interferometers are completing a $\simeq 2$ yr long ‘science run’ at design sensitivity, with GEO and Virgo on-line for part of the data-taking period. Merger-rate estimates are quite uncertain and for BH-NS binaries current detection-rate estimates reach as high as 0.2 yr^{-1} (e.g. O’Shaughnessy et al. 2007) for first-generation instruments. Work is already well underway for a first moderate upgrade of the two 4 km LIGO detectors leading to a new year-long science run in 2009 (enhanced LIGO/Virgo), before the major upgrade to Advanced LIGO/Virgo planned for the time-frame 2011–2014. These two upgrades are expected to increase detection rates by factors of about ~ 8 and 10^3 , respectively.

Looking beyond the first detections, the astrophysical analysis of signals and the measurements of inspiral source properties hold major promise for contributions to our understanding of outstanding astrophysical questions: e.g., the formation and evolution of black holes in close binaries and the origin of short-duration gamma-ray bursts (e.g. Nakar 2007). Such analysis will crucially rely on reliable methods for source parameter estimation applicable to a wide range of binary properties, signal strengths and number of instruments in the detector network.

Parameter estimation for binary inspiral signals is a challenging problem because of the large number of parameters (> 10) and the presence of strong correlations among some of them leading to a highly structured parameter space. These issues are further amplified in the case of compact objects with significant spins, as expected astrophysically for black holes especially (O’Shaughnessy et al. 2005; Belczynski et

al. 2007). Spins affect gravitational waveforms with both phase and amplitude modulations due to relativistic spin-orbit and spin-spin couplings (e.g. Apostolatos et al. 1994; Kidder 1995). Such effects are most prominent for high spin magnitudes and large angles between spins and orbital angular momentum in binaries with a high mass ratio (e.g. BH-NS). They can lead to significant loss of signal-to-noise ratio if non-spinning inspiral templates are used in the searches (Apostolatos et al. 1994, 1995; Grandclément et al. 2003; Buonanno et al. 2003) and in fact a major effort has been devoted to the development of efficient search algorithms that can circumvent the problem of high-dimensionality of the parameter space while maintaining high detection efficiency (Apostolatos 1996; Grandclément et al. 2003, 2004; Buonanno et al. 2003, 2004, 2005; Pan et al. 2004; Abbot et al. 2007). On the other hand, as we show here, the presence of spins benefits source-parameter estimation through the signal modulations, although still presenting us with a considerable computational challenge. This has already been highlighted in the context of LISA observations (see Vecchio 2004; Lang & Hughes 2006) but no study has been devoted so far to ground-based observations.

In this *Letter* we examine for the first time the potential for parameter estimation of spinning inspiral binaries with ground-based interferometers. We focus on BH-NS binaries with various spinning properties, since spin effects are strongest for these binaries (Apostolatos et al. 1994) and at the same time we are justified to ignore the NS spin, given the currently known NS spin periods even for recycled pulsars (e.g. Lorimer 2005). We employ a newly developed Markov-Chain Monte-Carlo (MCMC) algorithm applied on spinning inspiral signals injected into synthetic ground-based data and we derive posterior probability-density functions (PDFs) of all twelve signal parameters. We show that spin modulations allow us to constrain the source location and distance to some degree (in addition to mass and spin parameters), even with just one detector; this is in stark contrast to the case of non-spinning inspirals. For the more realistic case of even just two detectors, we show that quantitatively interesting constraints on astrophysical source parameters can be obtained with important implications for our understanding of BH formation, mass and spin evolution, as well as of short gamma-ray bursts.

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2. SIGNAL AND OBSERVABLES

In this Letter we concentrate on the signal produced during the in-spiral phase of two compact objects of mass $m_{1,2}$ and spins $\mathbf{S}_{1,2}$ in circular orbit. We focus on a fiducial BH-NS binary system with $M_1 = 10M_\odot$ and $M_2 = 1.4M_\odot$, so that we can ignore the NS spin $|\mathbf{S}_2| \ll |\mathbf{S}_1|$. Spins produce relativistic coupling of the angular momenta, leading to amplitude and phase modulation of the observed radiation due to the precession of the orbital plane during the observation time. Here we model GWs at the *restricted* post^{1.5}-Newtonian order and precession in the *simple-precession* limit (Apostolatos et al. 1994), which is appropriate for the unequal-mass system considered here. For sake of simplicity (it speeds up the waveform calculation considerably), we also ignore the contribution of the so-called *Thomas precession phase* (Apostolatos et al. 1994) to the total phase of the signal. In this simple-precession approximation, the orbital angular momentum \mathbf{L} and total spin $\mathbf{S} = \mathbf{S}_1 + \mathbf{S}_2 \simeq \mathbf{S}_1$ precess with the *same* angular frequency $\Omega_p = d\alpha/dt$ around a fixed direction $\hat{\mathbf{J}}_0 = \hat{\mathbf{J}} - \epsilon \hat{\mathbf{J}} \times \hat{\mathbf{L}} \approx \hat{\mathbf{J}}$, where $\mathbf{J} = \mathbf{L} + \mathbf{S}$ and $\epsilon \equiv \frac{L}{j} \frac{\dot{L}/L}{\Omega_p} \ll 1$; α is the angle traced by both \mathbf{L} and \mathbf{S} on the precession cone. During the inspiral phase the angle $\theta_{\text{SL}} \equiv \arccos(\hat{\mathbf{S}} \cdot \hat{\mathbf{L}})$ and $S = |\mathbf{S}|$ are constant. Such approximated waveforms have the advantage of retaining (at the leading order) all the salient qualitative features introduced by the spins, while allowing us to compute the waveforms analytically, which is greatly advantageous in terms of computational speed. While this approach is justified for exploration of GW astronomy and development of algorithms devoted to parameter estimation, more accurate waveforms (e.g. Kidder 1995; Faye et al. 2006; Blanchet et al. 2006) will be necessary for the analysis of real signals.

A binary system with one spinning compact object is described by a 12-dimensional parameter vector $\vec{\lambda}$. With respect to a fixed geocentric coordinate system our choice of independent parameters is:

$$\vec{\lambda} = \{d_L, \text{R.A.}, \text{Dec.}, \theta_{J_0}, \phi_{J_0}, \mathcal{M}, \eta, a_{\text{spin}}, \theta_{\text{SL}}, t_c, \phi_c, \alpha_c\}, \quad (1)$$

where $\mathcal{M} = (m_1 m_2)^{3/5} / (m_1 + m_2)^{1/5}$ and $\eta = m_1 m_2 / (m_1 + m_2)^2$ are the chirp mass and symmetric mass ratio, respectively; R.A. (right ascension) and Dec (declination) identify the source position in the sky $\hat{\mathbf{N}}$; the polar angles θ_{J_0} and ϕ_{J_0} – defined in the range $\theta_{J_0} \in [0, \pi[$ and $\phi_{J_0} \in [0, 2\pi[$ – identify the unit vector $\hat{\mathbf{J}}_0$; d_L is the luminosity distance to the source and $0 \leq a_{\text{spin}} \equiv S/m^2 \leq 1$ is the dimensionless spin magnitude; ϕ_c and α_c are integration constants that specify the GW phase and the location of \mathbf{S} on the precession cone, respectively, at a reference time that is chosen to be the time of coalescence t_c .

Given a network comprising n_{det} detectors, the data collected at the a -th instrument ($a = 1, \dots, n_{\text{det}}$) is given by $x_a(t) = n_a(t) + h_a(t; \vec{\lambda})$, where $h_a(t; \vec{\lambda}) = F_{a,+}(t)h_{a,+}(t; \vec{\lambda}) + F_{a,\times}(t)h_{a,\times}(t; \vec{\lambda})$ is the GW strain at the detector and $n_a(t)$ the detector noise. The astrophysical signal is given by the linear combination of the two independent polarisations $h_{a,+}(t; \vec{\lambda})$ and $h_{a,\times}(t; \vec{\lambda})$ weighted by the *time-dependent* antenna beam patterns $F_{a,+}(t)$ and $F_{a,\times}(t)$. An example of h_a for $\theta_{\text{SL}} = 55^\circ$ and $a_{\text{spin}} = 0.1$ and 0.8 is shown in panels a–b of Fig. 1. In our analysis we model the noise in each detector as a zero-mean Gaussian and stationary random process with one-sided noise spectral density $S_a(f)$ at the initial LIGO design sensitivity, where f is the frequency.

3. PARAMETER ESTIMATION: METHODS AND RESULTS

The goal of our analysis is to determine the *posterior* PDF of the unknown parameter vector $\vec{\lambda}$, Eq. (1) given the data sets x_a collected by a network of n_{det} detectors and the *prior* $p(\vec{\lambda})$ on the parameters. Bayes' theorem provides a rigorous mathematical rule to assign such a probability:

$$p(\vec{\lambda}|x_a) = \frac{p(\vec{\lambda})\mathcal{L}(x_a|\vec{\lambda})}{p(x_a)}; \quad (2)$$

in the previous Equation

$$\mathcal{L}(x_a|\vec{\lambda}) \propto \exp \left\{ -\frac{2}{\Delta} \sum_{f_j=f_i}^{f_h} \frac{|\tilde{x}_a(f_j) - \tilde{h}_a(f_j; \vec{\lambda})|^2}{S_a(f_j)} \right\} \quad (3)$$

is the *likelihood function* of the data given the model, which quantifies how the belief of the model is affected by the new observations and $p(x_a)$ is the *marginal likelihood* or *evidence*; $\tilde{x}_a(f_j)$ stands for the Fourier component of $x(t)$ at the discrete frequency f_j computed over a data segment of length Δ . For multi-detector observations involving a network of detectors with uncorrelated noise – this is the case of this paper, where we do not use the pair of co-located LIGO instruments at Hanford, WA – we have $p(\vec{\lambda}|x_a; a = 1, \dots, n_{\text{det}}) = \prod_{a=1}^{n_{\text{det}}} p(\vec{\lambda}|x_a)$.

The numerical computation of the joint and *marginalised* PDFs involves the evaluation of integrals over a large number of dimensions. Markov-Chain Monte-Carlo (MCMC) methods (e.g. Gilks et al. 1996; Gelman et al. 1997; and references therein) have proved to be particularly effective in tackling these numerical problems. We have developed an adaptive (see Figueiredo & Jain, 2002; Atchade & Rosenthal 2003) MCMC algorithm that can run multiple serial chains, intended to explore the parameter space efficiently while requiring the least amount of tuning for the specific signal at hand; the code is an extension of the one developed by some of the authors to explore MCMC methods for non-spinning binaries (Röver et al. 2006, 2007) and takes advantage of techniques explored by some of us in the context of LISA data analysis (Stroer et al. 2007); technical details will be provided elsewhere (Van der Sluys et al., in preparation).

Here we present results obtained by adding a signal in simulated initial-LIGO noise (at design sensitivity) and computing the posterior PDFs with MCMC techniques for a fiducial source consisting of a $10M_\odot$ spinning BH and a $1.4M_\odot$ non-spinning NS in a binary system at a distance of 13 Mpc. We consider a number of cases for which we change the BH spin magnitude ($a_{\text{spin}} = 0.1, 0.5, 0.8$) and the angle between the spin and the orbital angular momentum ($\theta_{\text{SL}} = 20^\circ, 55^\circ$); the remaining ten parameters, including source position and orientation of the total angular momentum, are kept constant (R.A. = 16.4h, Dec. = 40°, $\theta_{J_0} = 15^\circ$ and $\phi_{J_0} = 125^\circ$). For each of the six ($a_{\text{spin}}, \theta_{\text{SL}}$) combinations, we run the analysis using the data from (i) only one of the 4-km LIGO detectors ($n_{\text{det}} = 1$) and (ii) the two LIGO 4-km interferometers ($n_{\text{det}} = 2$). This results in a total of 12 signal cases explored in this study. The MCMC analysis that we carry out on each data set consists of 10 separate serial chains, each with a length of 3×10^7 iterations ($n_{\text{det}} = 1$) or 1.5×10^7 iterations ($n_{\text{det}} = 2$), sampled after a *burn-in* period (see e.g. Gilks et al. 1996) of 2×10^6 (1 detector) or 10^6 (2 detectors) samples. We start each chain at the true parameter values in order to minimize the computation time down to ≈ 10 days on a single 2.8 GHz CPU. Starting the chains from values significantly different

from the true values still allows us to sample the PDFs, but increases the computational cost by a factor of ~ 2 . An example of the PDFs obtained for a signal characterised by $a_{\text{spin}} = 0.1$ and $\theta_{\text{SL}} = 55^\circ$ is shown in panels c–f of Fig. 1, for the cases of 1 and 2 detectors; the PDFs for M_1 and M_2 in Fig. 1d are constructed from those obtained for \mathcal{M} and η .

In order to evaluate the parameter-estimation accuracy we compute probability intervals; in Table 1 we report the 90%-probability interval for each of the parameters, defined as the smallest range for which the posterior probability of a given parameter to be in that range is 0.9. For the 144 marginalized PDFs considered here (ignoring the derived parameters M_1 and M_2), the true parameter values all lie within the 90% probability range with the exception of 10 cases, marked with asterisks in Table 1. For those the true parameter is within the 95% (6 cases, marked with one asterisk in Table 1) and 99% (4 cases, marked with two asterisks in Table 1) probability interval.

From a conceptual point of view, the first key result is that even a single laser interferometer becomes a *pointing* instrument in observations of binaries with spinning BHs; it is able to constrain both the source sky location and its luminosity distance. This is in stark contrast to the case of binaries with non-spinning objects for which at least 3 instruments are necessary to resolve the sky location (although an ambiguity still remains for the two positions on the celestial sphere symmetric with respect to the plane containing the three interferometers; see e.g. Jaranowski, P., & Krolak 1994; Pai et al. 2001; Cavalier et al. 2006; Röver et al. 2007; and references therein). This is due to the fact that the presence of the BH spin breaks degeneracies among parameters that encode the information of the geometry of the source and affects the amplitude and polarization-phase modulations. For observations with a single interferometer, the accuracy of parameter estimation is rather poor, see Fig. 1c–f and Table 1. However for observations carried out with even just two of the LIGO instruments, *all* the source parameters can be measured at astrophysically interesting levels, including distance, individual masses, spin magnitude and tilt angle that are possibly outside the reach of electromagnetic telescopes; the typical GW error box is $\simeq 3^\circ \times 10^\circ$ and the luminosity distance can be measured with a $\simeq 30\%$ error; individual masses can be estimated with a $\simeq 10\%$ error and spin magnitudes and opening angles with a $\simeq 50\%$ error.

As expected, the parameter-estimation accuracy depends strongly on the actual spin parameters of the system in particular a_{spin} and θ_{SL} (see Table 1): as a general trend, the larger a_{spin} and θ_{SL} , the stronger the modulations and the signal-to-noise ratio, which increases the ability of measuring the source parameters. However, the complex structure of the likelihood function, and correlations amongst different parameters, play also an important role; the width of the 90%-probability interval is in fact not strictly monotonic as a function of a_{spin} and θ_{SL} . Not surprisingly, the other main factor that affects parameter determination is the number of detectors in the network, a result well established in studies of inspirals of non-spinning compact objects (e.g. Jaranowski, P.,

& Krolak 1994; Pai et al. 2001; Cavalier et al. 2006; Röver et al. 2007). We note that a dependence on the other parameters, describing the source’s position in the sky and the (initial) orientation of the orbital angular momentum, are not explored in this initial study.

4. CONCLUSIONS

We have explored for the first time the potential of astronomical observations of binary systems with spinning compact objects. As a fiducial source we have considered a BH-NS system with a spinning BH undergoing ‘simple precession’ and we have modeled the gravitational radiation at the restricted post^{1.5}-Newtonian order. We have shown that in the presence of spins even a single detector becomes a pointing instrument and that two interferometers, such as the pair of the transcontinental 4-km LIGO interferometers, can constrain the source location with an error $\simeq 10^\circ$ and measure the relevant source parameters, such as distance, individual masses and spin at the level of several tens of percent or better. Such information could not be extracted in the case of non-spinning objects and would require three or more instruments in coincident operation. Moreover, the direct determination of these key parameters is notoriously difficult in the case of electromagnetic observations. These performances, coupled with timing resolution from GW observations in the range $\approx 3\text{--}20$ ms, could allow the identifications of electromagnetic counterparts if associated with binary compact-object mergers and provide essential information on outstanding problems connected to short gamma-ray bursts and the formation and evolution of BH compact binaries.

The analysis presented in this *Letter* is the first step of a more detailed study that we are currently carrying out, exploring a much larger parameter space, developing techniques to reduce the computational cost of these simulations, and testing the methods with actual LIGO data. The waveform model, though adequate for exploratory studies, is not sufficiently accurate for the analysis of real data, and we plan to consider more realistic waveforms in the future. Other instruments, such as Virgo and GEO are operating in coincidence with LIGO; including data also from those detectors (straightforward in our approach) will improve the quality of GW astronomy, in particular angular resolution and distance measurements as well as mass and spin parameters. Finally, we intend to further develop our Bayesian approach into one of the standard tools that can be included in the analysis pipeline used for the processing of the ‘science data’ collected by ground-based laser interferometers.

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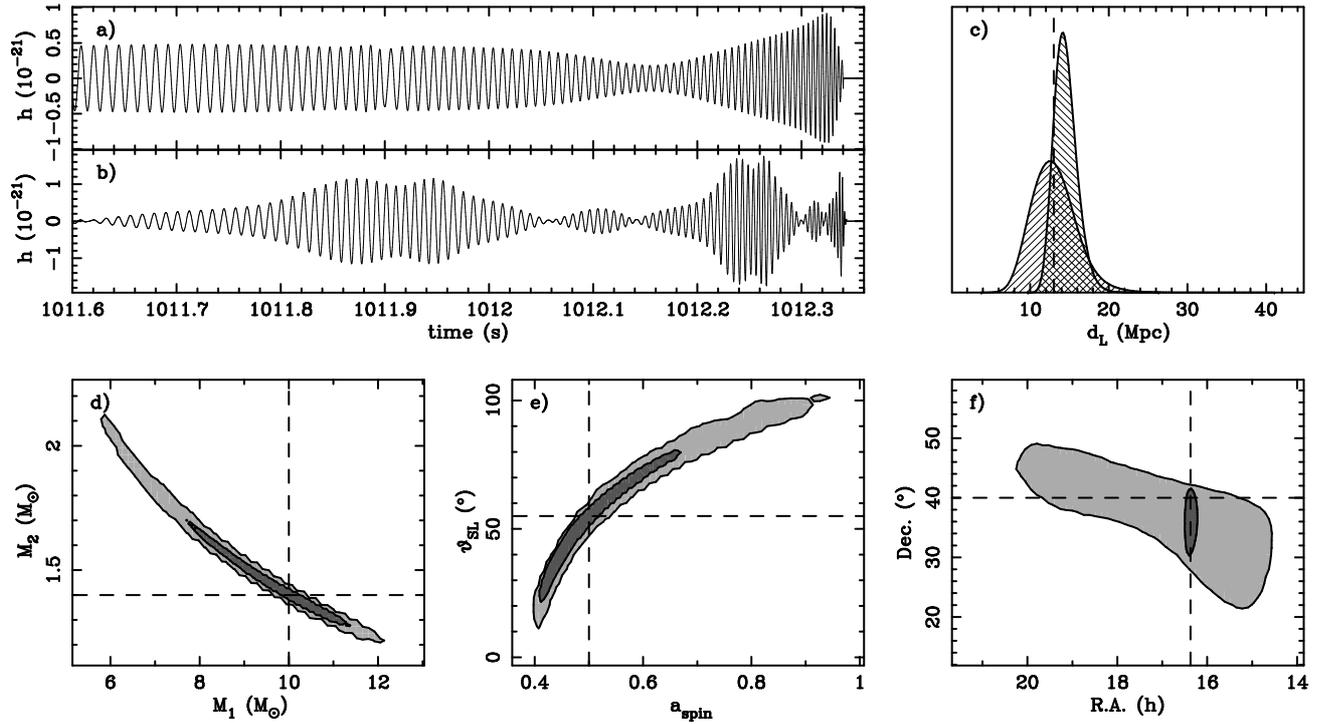


FIG. 1.— (a) Part of the waveform from a source with $a_{\text{spin}} = 0.1$ and $\theta_{\text{SL}} = 55^\circ$. (b) the same waveform, but for $a_{\text{spin}} = 0.8$. (c) Posterior PDF of the luminosity distance for a signal with $a_{\text{spin}} = 0.5$ and $\theta_{\text{SL}} = 55^\circ$, as determined with the signal of one (broad PDF) and two (narrow PDF) 4-km LIGO detectors. The dashed line shows the true distance. (d–f) Two-dimensional posterior PDF showing the results for the same runs as (c), for the chirp mass and symmetric mass ratio (d), the spin parameters (e) and the position in the sky (f). Light and dark shades show the result for one and two detectors respectively. The dashed lines display the true parameter values.

TABLE 1
SNRS AND WIDTHS OF THE 90%-PROBABILITY INTERVALS OF THE MCMC RUNS DESCRIBED IN THE TEXT.

n_{det}	θ_{SL} ($^\circ$)	a_{spin}	SNR	M_1 (M_\odot)	M_2 (M_\odot)	\mathcal{M} (M_\odot)	η (%)	t_c (s)	d_L (Mpc)	a_{spin} (%)	θ_{SL} ($^\circ$)	RA ^a ($^\circ$)	Decl. ($^\circ$)	ϕ_c ($^\circ$)	θ_{J_0} ($^\circ$)	ϕ_{J_0} ($^\circ$)	α_c ($^\circ$)
1	20	0.1	12.6	60	46	1.6	65	0.042	155	199	130*	224	133	323	130	310	318
1	20	0.5	13.7	18	14	0.61	24	0.009	117	14	29	58	26	318	19	73	148
1	20	0.8	14.7	33	25	0.62	39	0.011	75	15	30	40	20	317	20	48	288
1	55	0.1	12.3	26	19	0.56	33	0.017	64	130	95	52	31	322*	21	59	89
1	55	0.5	18.5	46	34	0.96	55	0.012	67	57	54	38	16	323**	13	50	292
1	55	0.8	20.5	34	24	1.1	43	0.010	55	34	39	30	12	323	12	36	322
2	20	0.1	17.3	41	31	0.98	50	0.017	53	144	135	31	30*	322	33*	23	177
2	20	0.5	20.1	6.8**	5.2**	0.27*	8.8**	0.004	27	6.4	17	2.5	11	210*	12	7.6	40**
2	20	0.8	21.6	9.5	7.2	0.39	13	0.004	31	6.9	15	2.7	12	305	12	8.6	245
2	55	0.1	18.2	8.0	6.1	0.44	11	0.005	31	95	48	3.1	13	301	12	7.9	59
2	55	0.5	27.5	20	15	0.58	26	0.003	32	28	36	1.8	5.9**	320	9.5	8.5	269
2	55	0.8	30.5	29	21	0.71	37	0.003	33	29	25	1.8	5.6	314	6.9	9.1	319

^a The numbers cited are the width of the R.A. PDFs multiplied with $\cos(40^\circ)$ (where 40° is the declination of the source) and expressed in degrees to make them comparable to the declination width. For entries marked with *, the true value lies outside the 90%-probability range, for those marked with ** it lies outside the 95%-probability range. All true values lie within the 99%-probability range.

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