Some Ideas on Coatingless all-reflective ITF

Adalberto Giazotto (*)

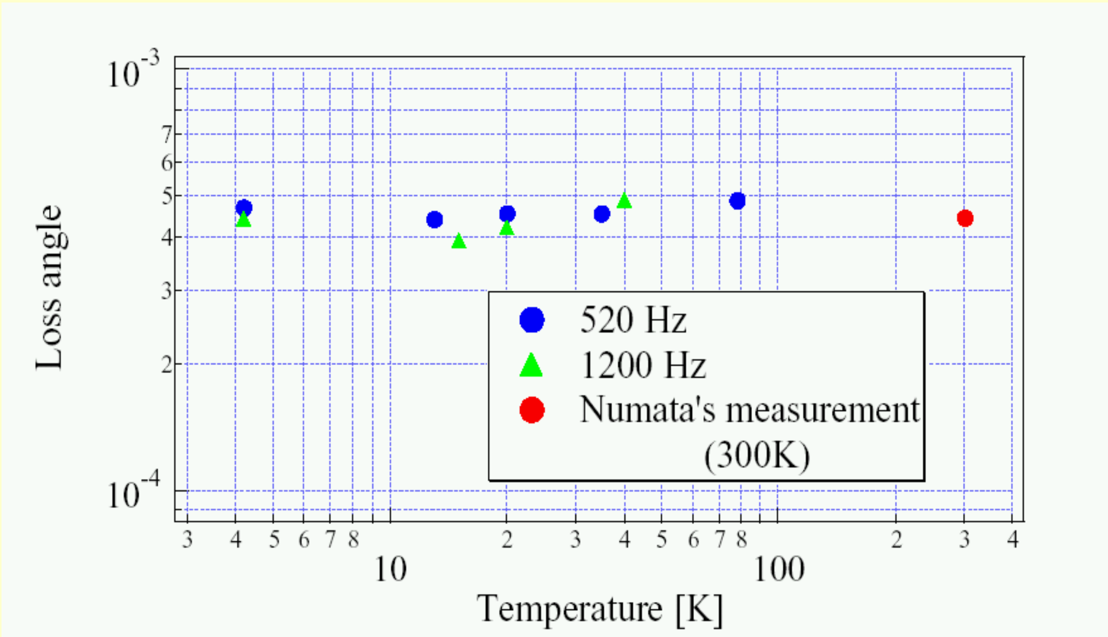
INFN- Pisa

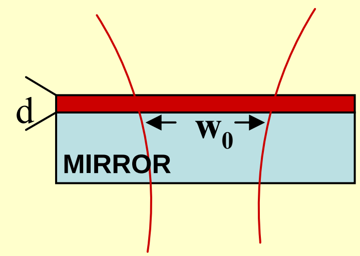
(*) Work done in collaboration with G. Cella

G0900644-v1

Why Total Reflecting Mirrors?

Experimental data from Yamamoto and Numata show a very high coating loss angle even at low temperature.





$E_C/E_B = d/w_0 \sim 10^{-4}$

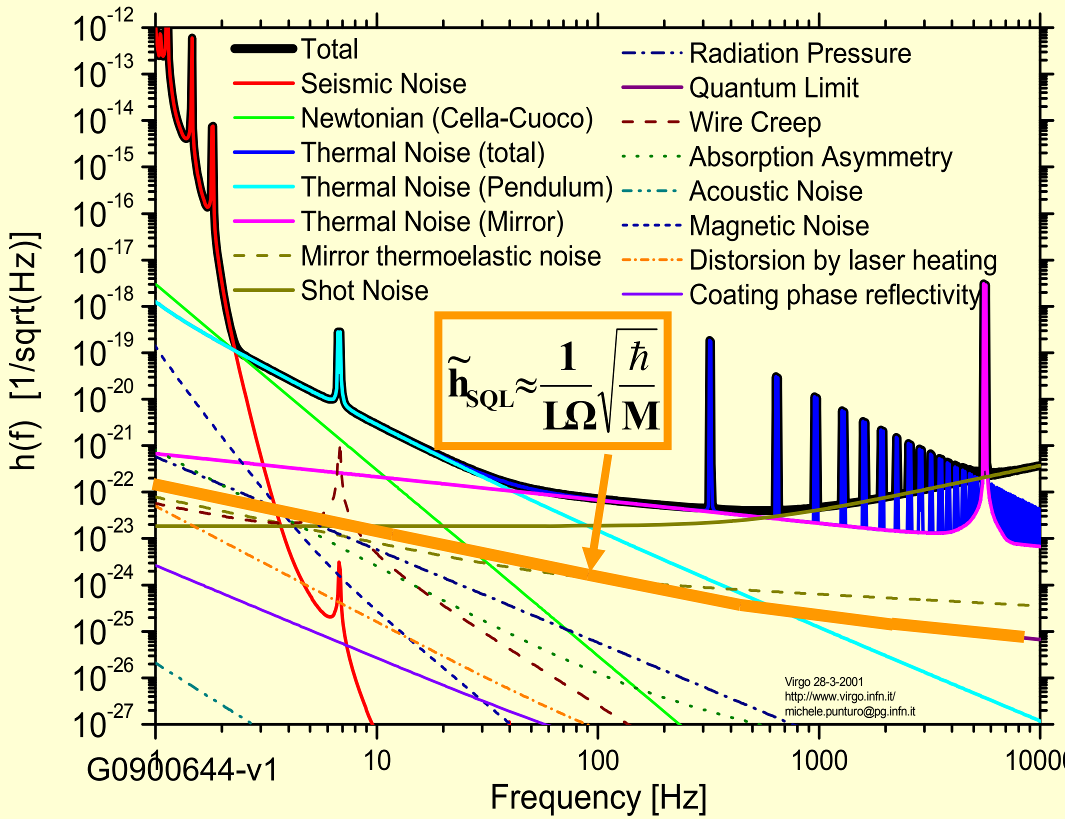
$$\widetilde{x}_{Total}(\omega) = \frac{\sqrt{16 k_B T (\phi_{Bulk} + \frac{E_C}{E_B}\phi_{Coating})}}{\omega^{1/2} E^{1/2} w_0^{1/2}}$$

$$\phi_{Total} = \phi_{Bulk} + \frac{E_C}{E_B}\phi_{Coating} \sim \frac{E_C}{E_B}\phi_{Coating} \sim 410^{-8}$$

$$Q = \frac{1}{\phi_{Total}} \sim 2.510^{7}$$

The Virgo design Thermal noise curve is evaluated with a Q~10^6, and the maximum expected Q, due to coating losses, is Q~2.5 10^7. Standard Quantum Limit is about a factor 100 below Virgo TN i.e. equivalent to a Q=10^{10} ; consequently, for reaching SQL sensitivity Q should improve by 10^{10}/ 2.5 10^7~ 400- For this reason it is interesting to explore Coating-less Mirrors

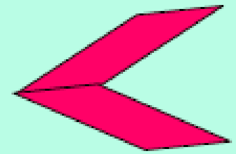
This argument will become even more important if we want to go below SQL.

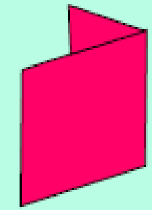


G0900644-v1

Some History

Toraldo di Francia in 1965 proposes flat Roof Prism for creating a stable Radio Frequency cavity. Stability was succesfully experimentally tested.





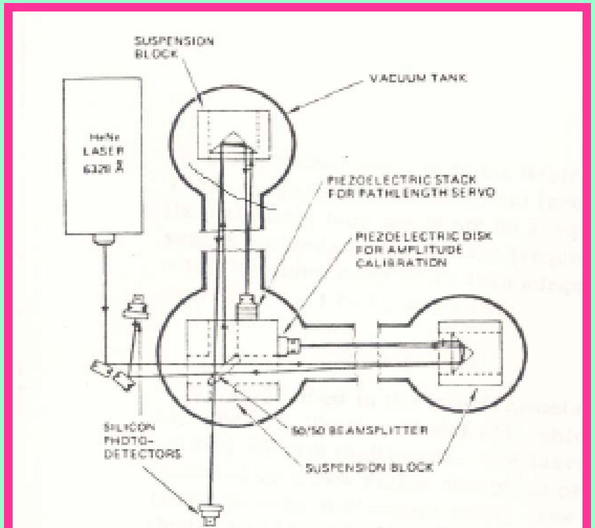
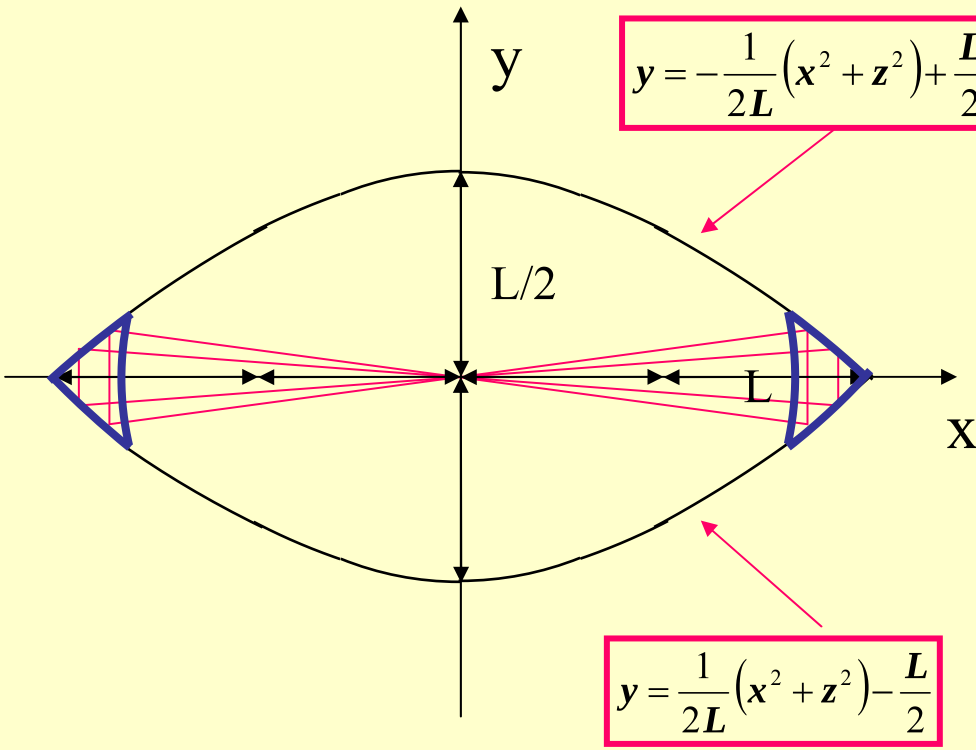


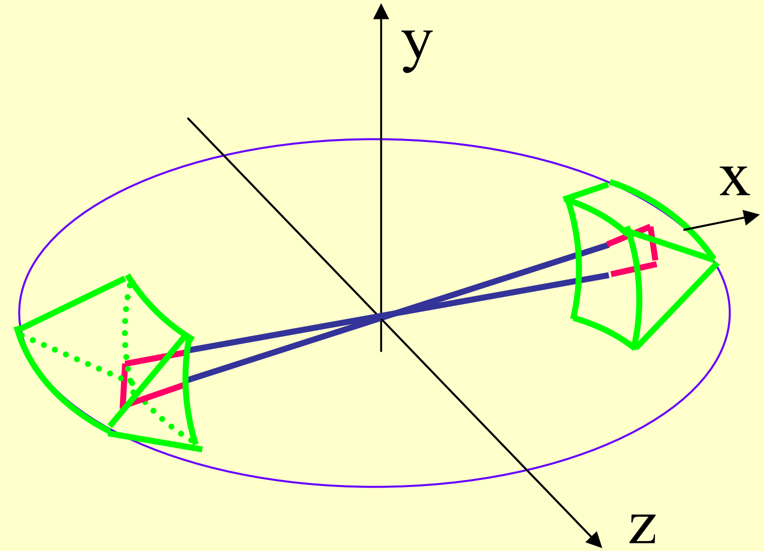
FIG. 7. Schematic of folded optical path.

In 1970 **Robert Forward** (Hughes Lab.) build the first Interferometer for GW detection. It was equipped with **Roof Prisms Mirrors.** Arm length ~2m

Braginsky et al. Recently Published a paper on the use of Roof Prism and Corner Cube mirrors in Fabry Perot cavities using Antireflective Coatings .

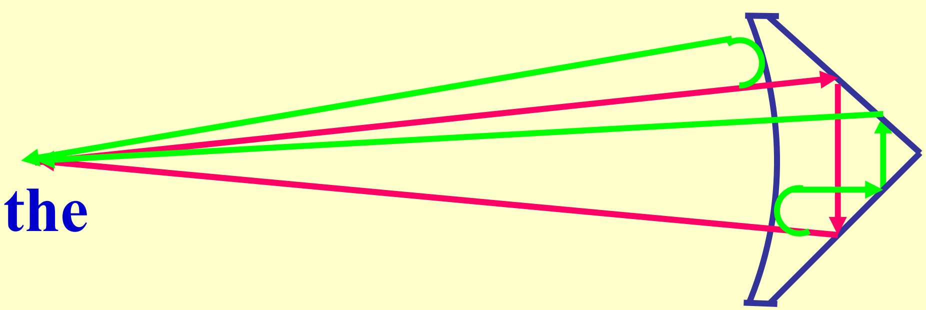
Rotation Parabolas as exact reflectors for closed geometrical optical trajectories





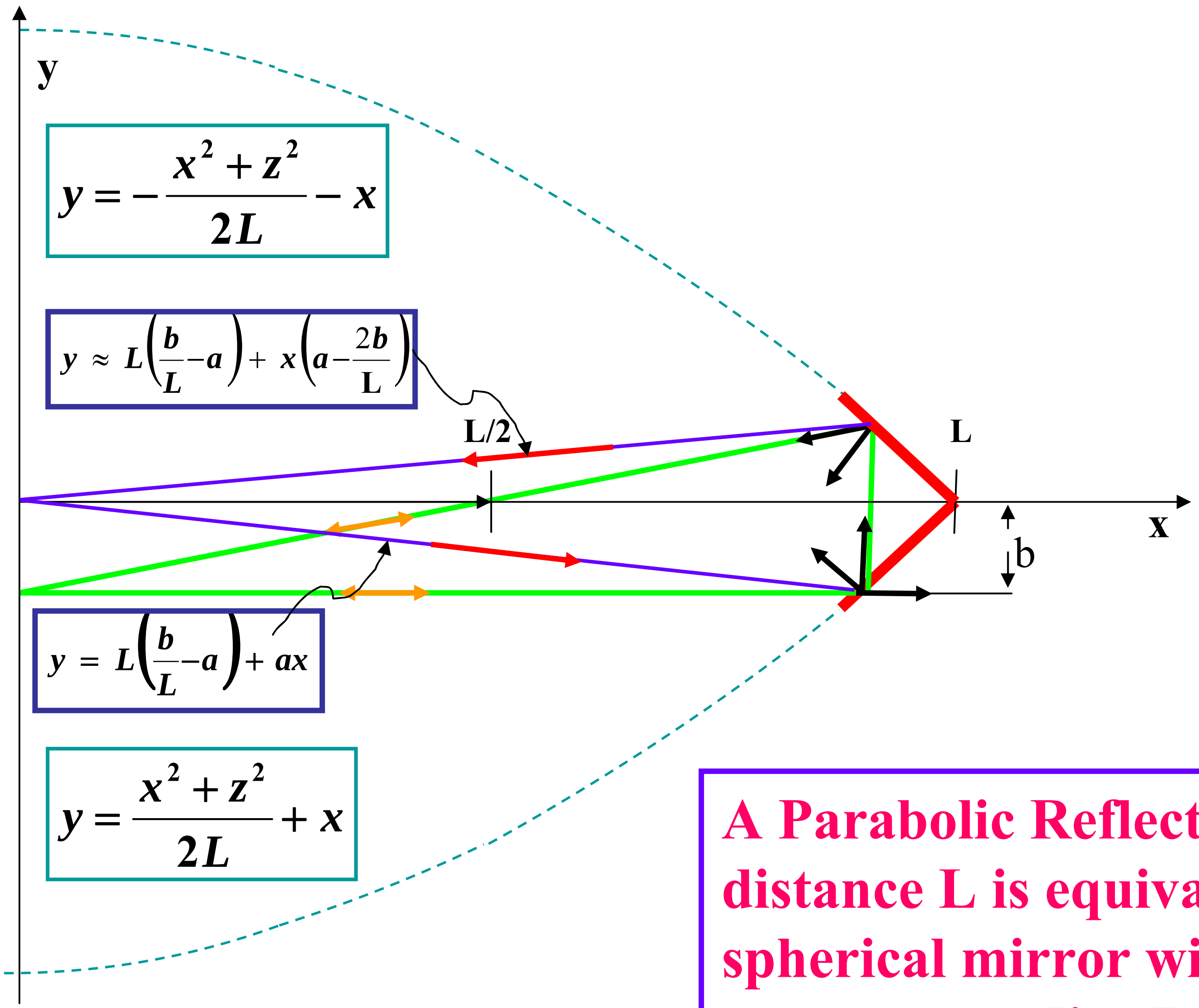
No Beam Losses

The spherical mirror surface is matched to the constant phase beam surface curvature



G0900644-v1

Rotation Parabolas as reflectors for closed geometrical optics trajectories



$$y = -\frac{x^2 + z^2}{2L} - x$$

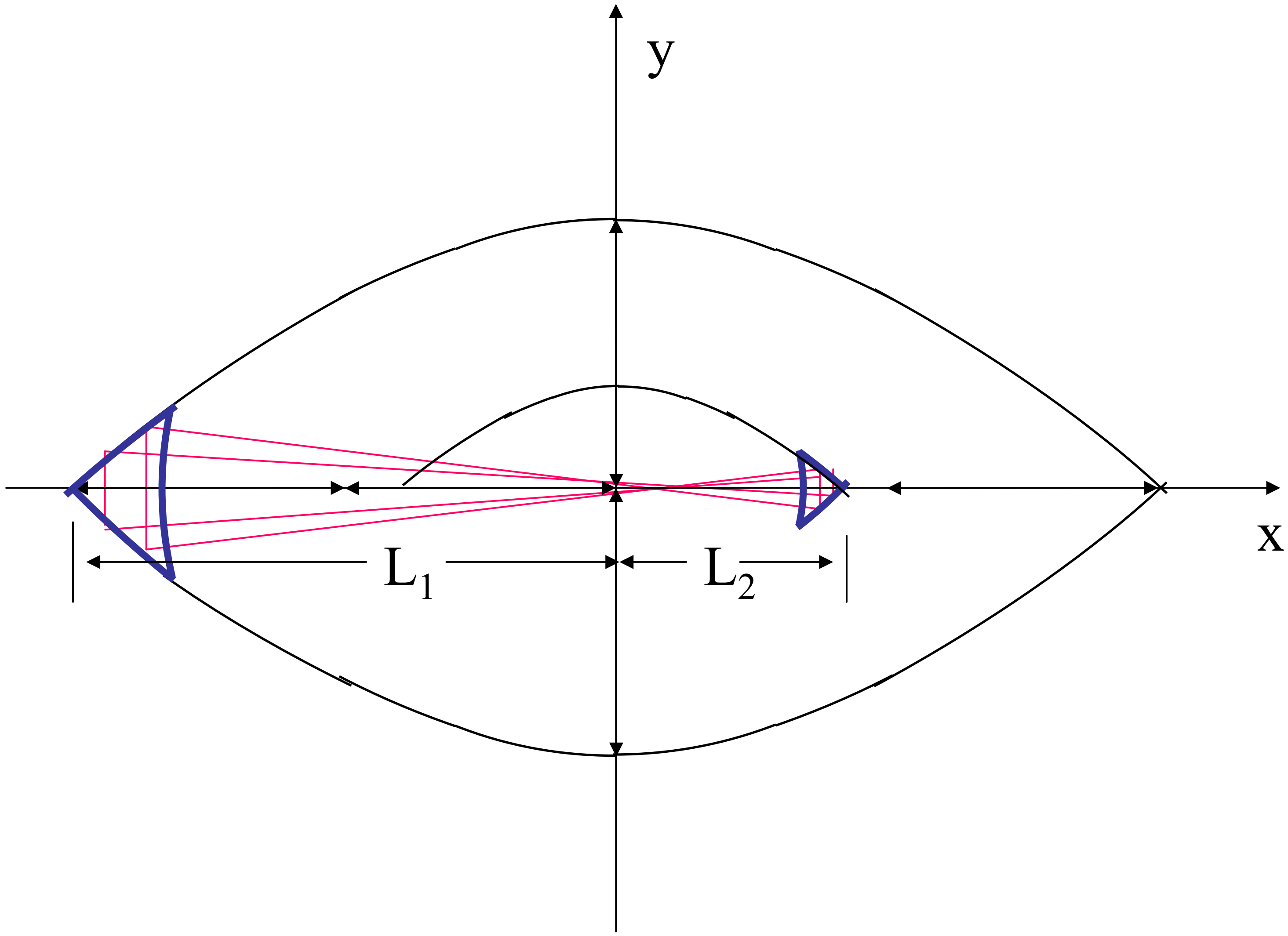
$$y \approx L\left(\frac{b}{L} - a\right) + x\left(a - \frac{2b}{\mathrm{L}}\right)$$

$$y = L\left(\frac{b}{L} - a\right) + ax$$

$$y = \frac{x^2 + z^2}{2L} + x$$

A Parabolic Reflector at distance L is equivalent to a spherical mirror with curvature radius L/2

Reflective cavity with Asymmetric Arms



Roof Prism curvature

$$y = -\frac{1}{2L}\left(x^2+z^2\right)+\frac{L}{2}$$

$$\begin{pmatrix} y \\ x \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}\begin{pmatrix} y' \\ x' \end{pmatrix} \Rightarrow \frac{1}{\sqrt{2}}\begin{pmatrix} y'-x' \\ y'+x'+\sqrt{2}L \end{pmatrix}$$

$$\begin{cases} y=-\frac{1}{2L}\left(x^2+z^2\right)+\frac{L}{2} \\ y=x+c \end{cases}$$

$$y'-x'=-\frac{\sqrt{2}}{2L}\left(\frac{\left(y'+x'+\sqrt{2}L\right)^2}{2}+z^2\right)+\frac{L\sqrt{2}}{2}=-\frac{\sqrt{2}}{2L}\left(\frac{y'^2+x'^2+2L^2+2\sqrt{2}Ly'+2\sqrt{2}Lx'+2x'+2y'}{2}+z^2\right)+\frac{L\sqrt{2}}{2}$$

$$y'-x'+\frac{\sqrt{2}}{2L}\left(\frac{y'^2+x'^2+2L^2+2\sqrt{2}Ly'+2\sqrt{2}Lx'+2y'x'}{2}+z^2\right)-\frac{L\sqrt{2}}{2}=0$$

$$y'^2\left(\frac{\sqrt{2}}{4L}\right)+y'\left(1+1+\frac{\sqrt{2}x'}{2L}\right)+\frac{\sqrt{2}}{4L}x'^2+\frac{\sqrt{2}L}{2}-\frac{L\sqrt{2}}{2}+\frac{\sqrt{2}}{2L}z^2=0$$

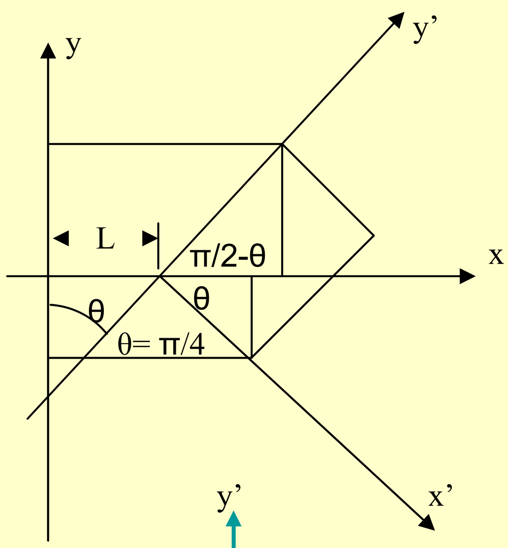
$$y'^2\left(\frac{\sqrt{2}}{4L}\right)+y'\left(2+\frac{\sqrt{2}x'}{2L}\right)+\frac{\sqrt{2}}{4L}x'^2+\frac{\sqrt{2}}{2L}z^2=0$$

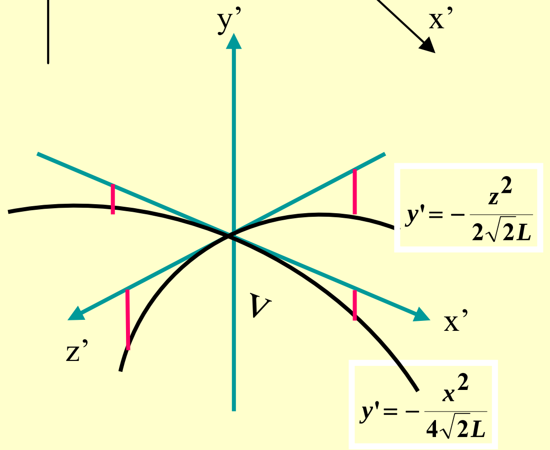
$$y'=\frac{-\left(2+\frac{\sqrt{2}x'}{2L}\right)\pm\sqrt{\left(2+\frac{\sqrt{2}x'}{2L}\right)^2-\frac{\sqrt{2}}{L}\left(\frac{\sqrt{2}}{4L}x'^2+\frac{\sqrt{2}}{2L}z^2\right)}}{\frac{\sqrt{2}}{2L}}=\frac{-\left(2+\frac{\sqrt{2}x'}{2L}\right)\pm\sqrt{4+\frac{2x'^2}{4L}+\frac{2\sqrt{2}x'}{L}-\frac{1}{L^2}\left(\frac{1}{2}x'^2+z^2\right)}}{\frac{\sqrt{2}}{2L}}$$

$$\left(1+\alpha\right)^{1/2}=1+\frac{1}{2}\left(1+\alpha\right)^{-1/2}\alpha-\frac{1}{4}\left(1+\alpha\right)^{-3/2}\frac{\alpha^2}{2}=1+\frac{\alpha}{2}-\frac{\alpha^2}{8}$$

$$y'=\frac{-\left(2+\frac{\sqrt{2}x'}{2L}\right)\pm 2\sqrt{1+\frac{\sqrt{2}x'}{2L}-\frac{z^2}{4L^2}}}{\frac{\sqrt{2}}{2L}}\approx\frac{-2-\frac{\sqrt{2}x'}{2L}\pm 2\left(1+\frac{\sqrt{2}x'}{4L}-\frac{z^2}{8L^2}-\frac{1}{8}\left(\frac{\sqrt{2}x'}{2L}-\frac{z^2}{4L^2}\right)^2\right)}{\frac{\sqrt{2}}{2L}}=\frac{-2-\frac{\sqrt{2}x'}{2L}\pm 2\left(1+\frac{\sqrt{2}x'}{4L}-\frac{z^2}{8L^2}-\frac{x'^2}{16L^2}\right)}{\frac{\sqrt{2}}{2L}}$$

$$y'=-\frac{z^2}{2\sqrt{2}L}-\frac{x'^2}{4\sqrt{2}L}=$$

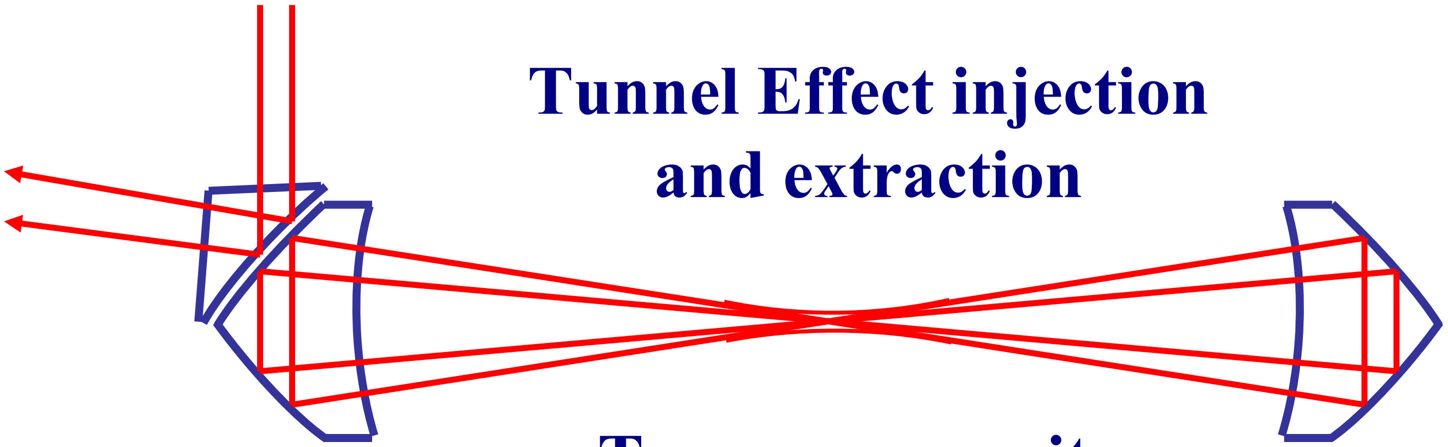




Cavity Beam Injection

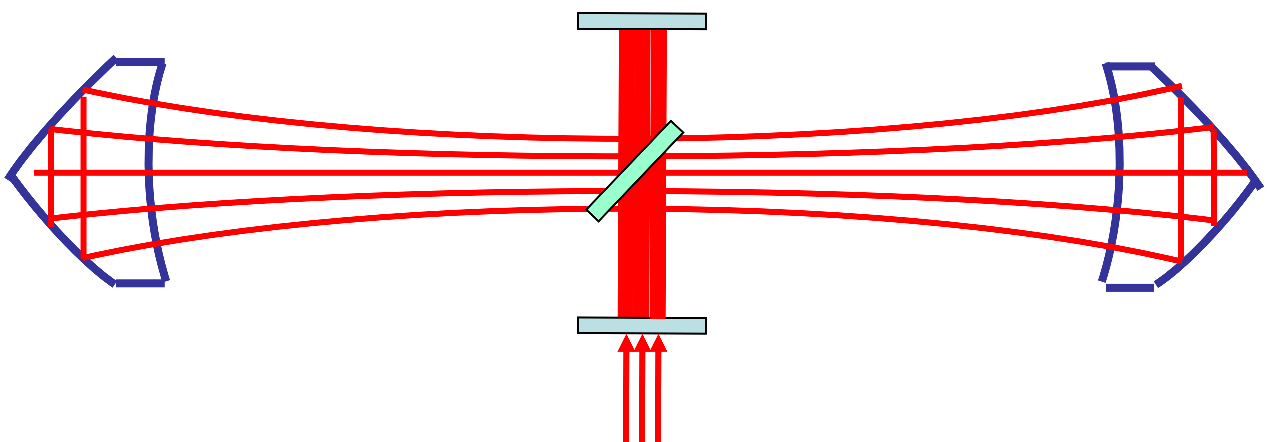
One of the main problems we faced was to find the way to inject a beam in the All Reflective Cavity.

Tunnel Effect injection and extraction



Parabolic Roof Prism

Transverse cavity injection and extraction

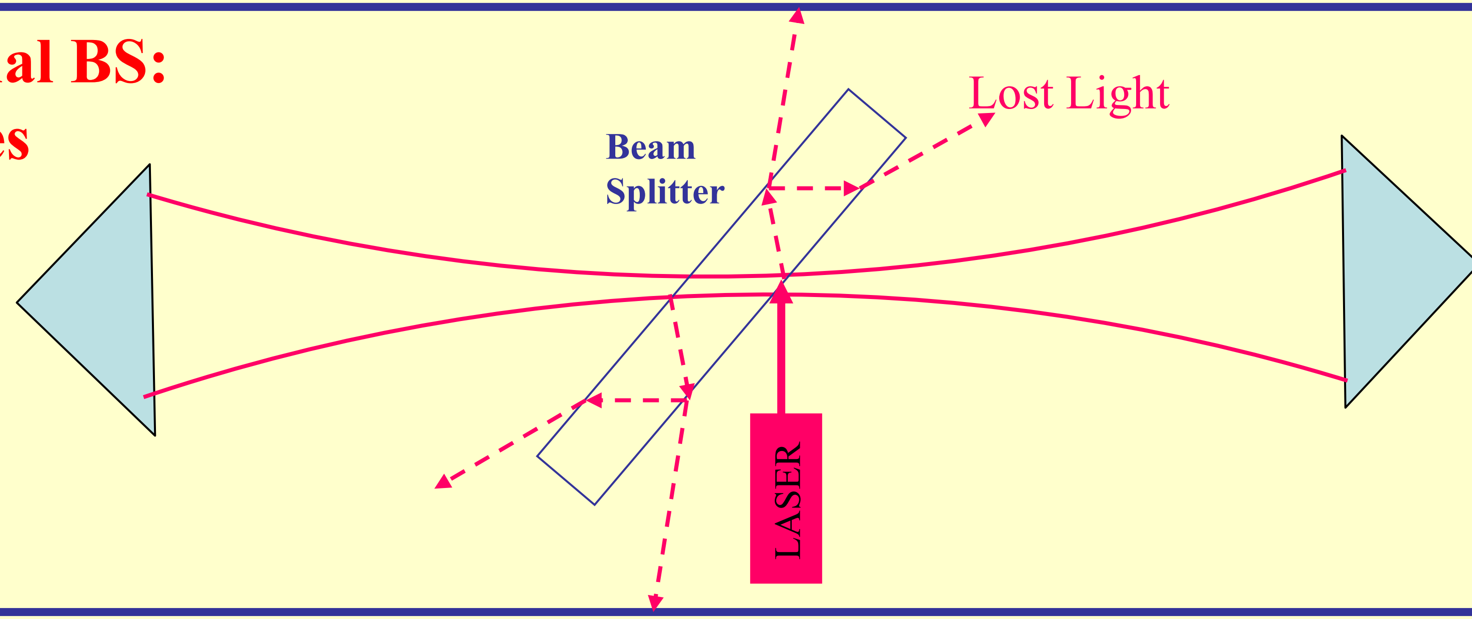


Corner Cube

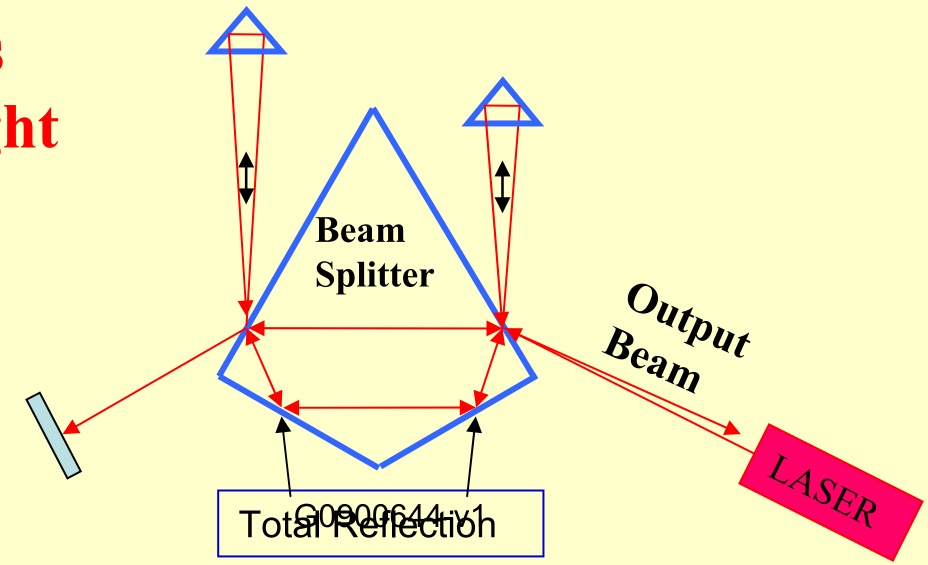
(V.Braginsky et al.)

Example of Power Injection in a Parabolic Coatingless all-Reflective Cavity

Conventional BS: Light Losses



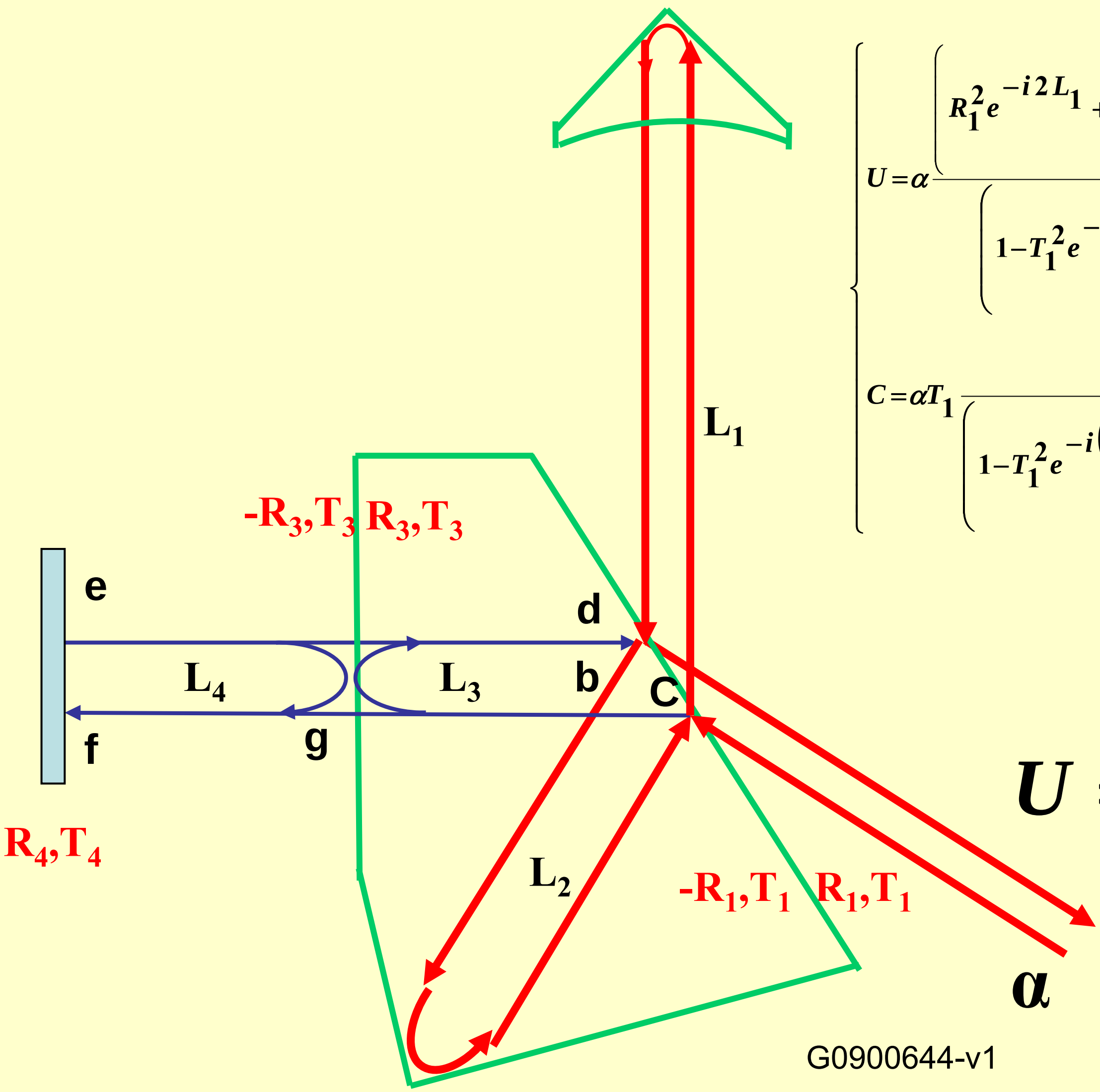
Closed trajectories inside BS : No Light Losses



A REALISTIC CAVITY

Coatingless, tunable finesse interferometer for gravitational wave detection

G. Cella and A. Giazotto-Phys. Rev D 74, 042001 (2006)



$$\begin{cases} U=\alpha\dfrac{\left(R_1^2e^{-i2L_1}+\left(T_1^2e^{-i2L_3}-e^{-i\left(2L_1+2L_2+2L_3\right)}\right)\dfrac{R_3+R_4e^{-2iL_4}}{1+R_4R_3e^{-i2L_4}}\right)}{\left(1-T_1^2e^{-i\left(2L_2+2L_1\right)}-R_1^2e^{-i\left(2L_2+2L_3\right)}\dfrac{R_3+R_4e^{-2iL_4}}{1+R_4R_3e^{-i2L_4}}\right)} \\ C=\alpha T_1\dfrac{\left(1-e^{-i\left(2L_2+2L_1\right)}\right)}{\left(1-T_1^2e^{-i\left(2L_2+2L_1\right)}-R_1^2e^{-i\left(2L_2+2L_3\right)}\dfrac{R_3+R_4e^{-2iL_4}}{1+R_4R_3e^{-i2L_4}}\right)} \end{cases}$$

$$U=e^{i\varphi}$$

G0900644-v1

Some Peculiar Properties-1

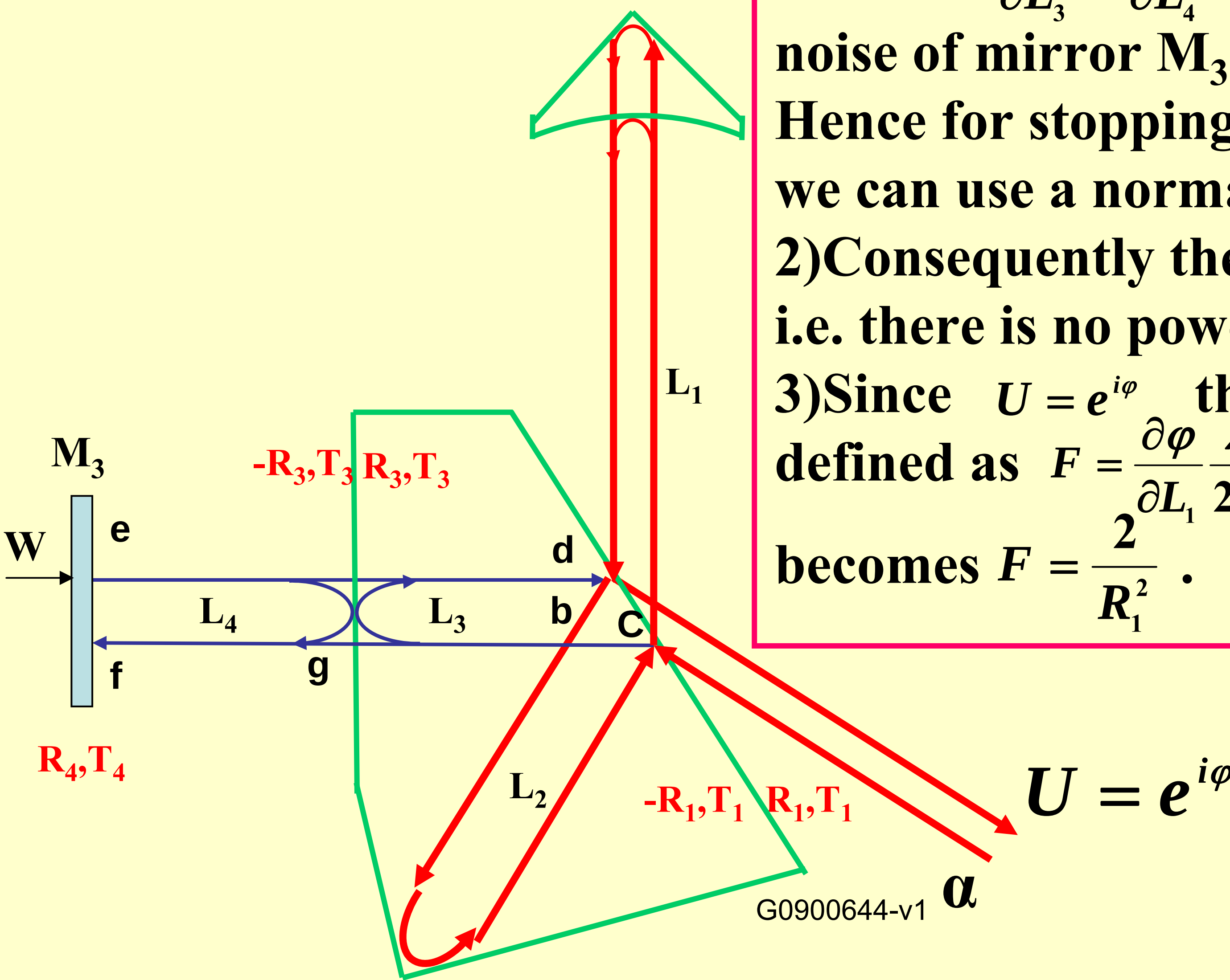
If channel L3-L4 is antiresonating

1)Then $\dfrac{\partial U}{\partial L_3} = \dfrac{\partial U}{\partial L_4} = 0$ **i.e. Thermal noise of mirror M_3 does not affect U. Hence for stopping input of vacuum we can use a normally coated mirror**

2)Consequently the amplitude C=0. i.e. there is no power on M_3

3)Since $U = e^{i\varphi}$ **the Finesse F, defined as** $F = \dfrac{\partial \varphi}{\partial L_1}\dfrac{\lambda}{2\pi} \Rightarrow -\dfrac{i}{U}\dfrac{\partial U}{\partial L_1}\dfrac{\lambda}{2\pi}$ **,**

becomes $F = \dfrac{2}{R_1^2}$ **.**



$$\delta\varphi = \frac{2}{R_1^2}\left(1+\frac{1-R_1^2}{1+R_1^2}\chi^2+\frac{\left(2+2R_1^2-5R_1^4+R_1^6\right)\chi^4}{3\left(1+R_1^2\right)^3}+O\left[\chi^5\right]\right)\frac{\omega_0}{c}\delta L_1$$

–

Some Peculiar Properties-2

If channel L_3-L_4 is out of antiresonance with a small phase offset χ then

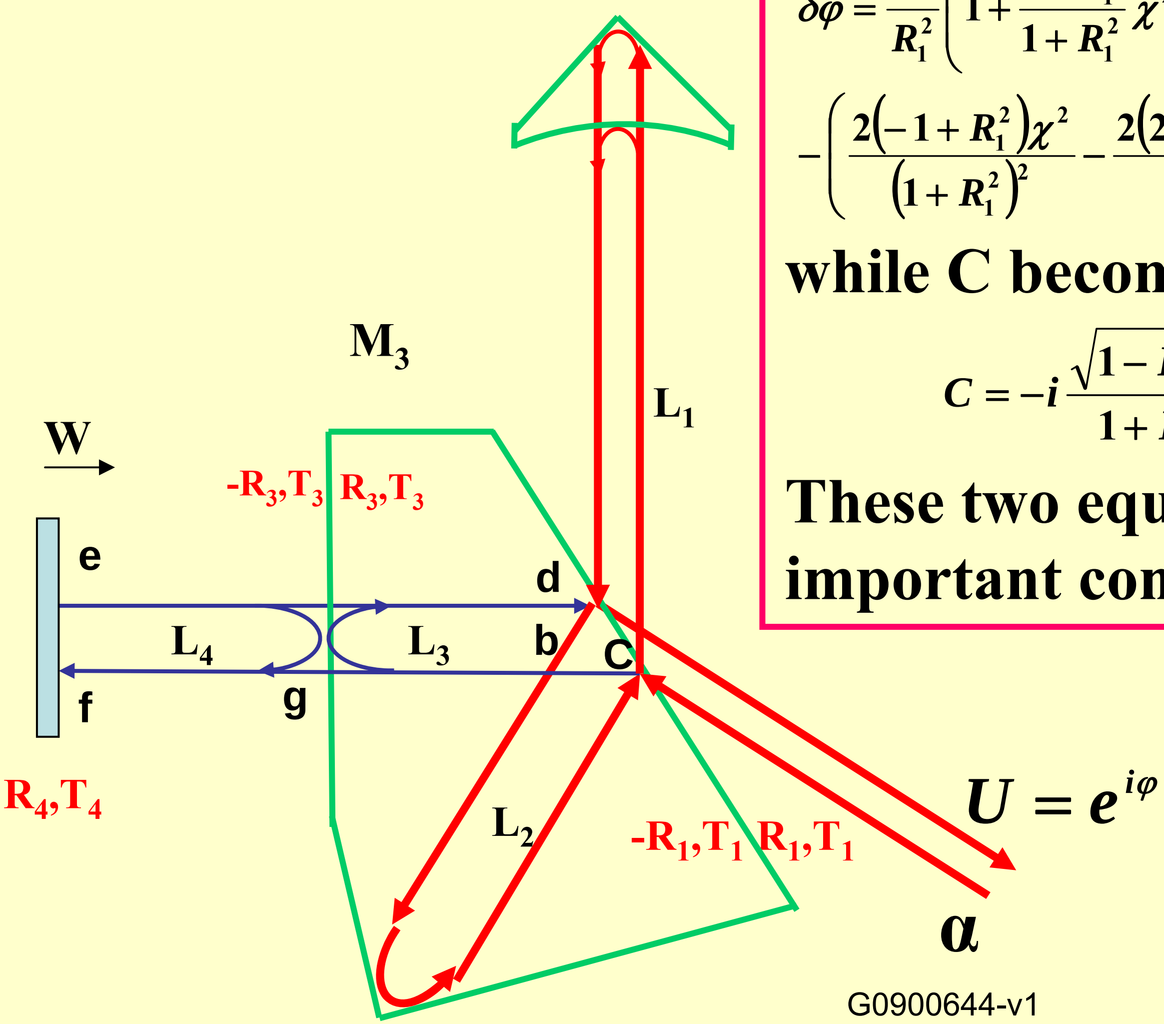
$$\delta\varphi = \frac{2}{R_1^2}\left(1+\frac{1-R_1^2}{1+R_1^2}\chi^2+\frac{\left(2+2R_1^2-5R_1^4+R_1^6\right)\chi^4}{3\left(1+R_1^2\right)^3}+O\left[\chi^5\right]\right)\frac{\omega_0}{c}\delta L_1$$

$$-\left(\frac{2\left(-1+R_1^2\right)\chi^2}{\left(1+R_1^2\right)^2}-\frac{2\left(2+8R_1^2-11R_1^4+R_1^6\right)\chi^4}{3\left(1+R_1^2\right)^4}+O\left[\chi^5\right]\right)\frac{\omega_0}{c}\delta L_4$$

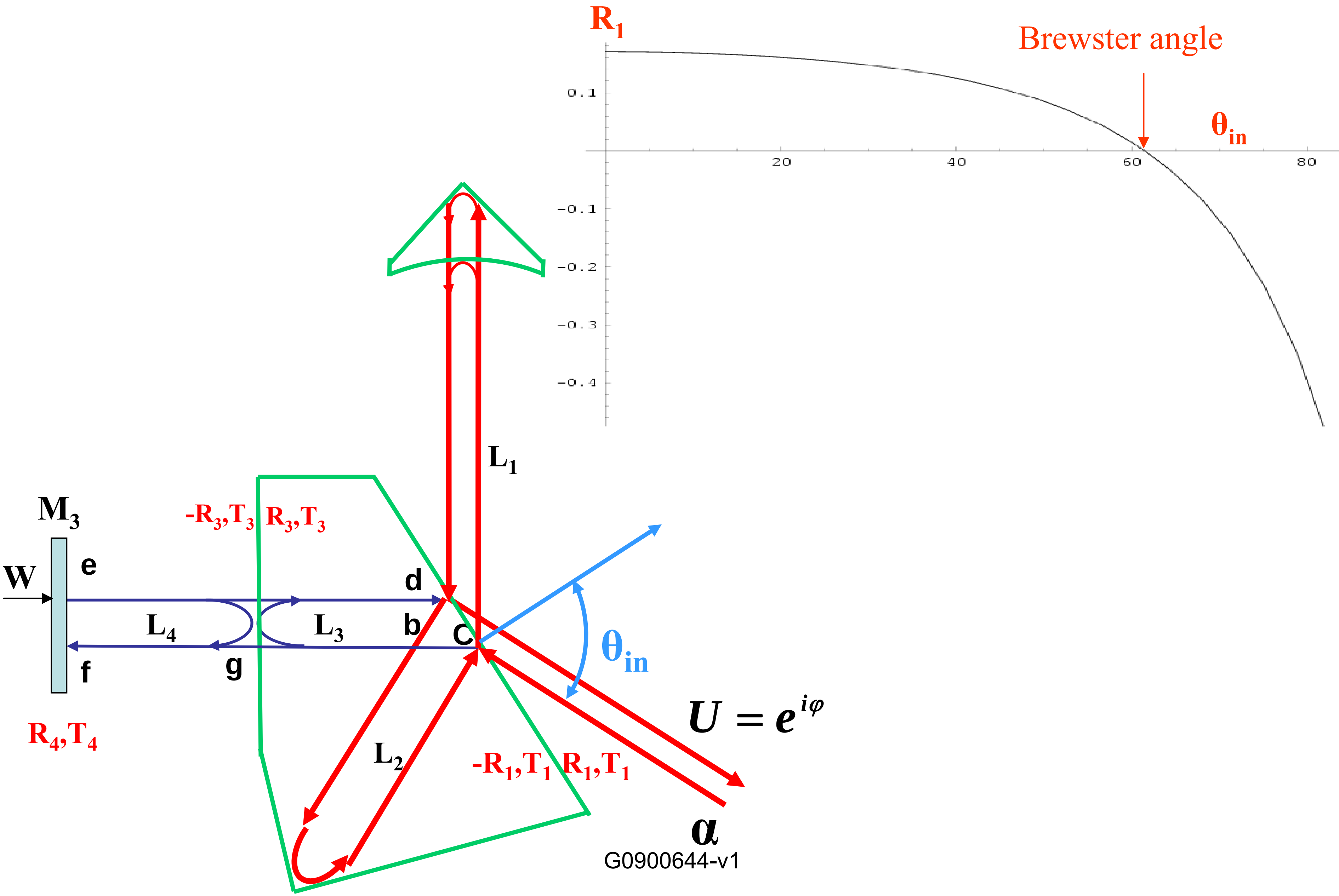
while C becomes

$$C = -i\frac{\sqrt{1-R_1^2}\,\chi}{1+R_1^2}-\frac{R_1^2\sqrt{1-R_1^2}\,\chi^2}{\left(1+R_1^2\right)^2}+O\left(\chi^3\right)$$

These two equations seem to have important consequences.



Silica Reflectivity



G0900644-v1

DISCUSSION

By considering that $R_1^2 \ll 1$, by stopping expansion to χ^2 terms and by putting in evidence $F = \frac{2}{R_1^2}$, we obtain:

$$\delta\varphi = \frac{2}{R_1^2}\left[\left(1+\chi^2\right)\frac{\omega_0}{c}\delta L_1 + R_1^2\chi^2\frac{\omega_0}{c}\delta L_4\right] = \frac{2\left(1+\chi^2\right)}{R_1^2}\left[\frac{\omega_0}{c}\delta L_1 + R_1^2\frac{\chi^2}{\left(1+\chi^2\right)}\frac{\omega_0}{c}\delta L_4\right]$$

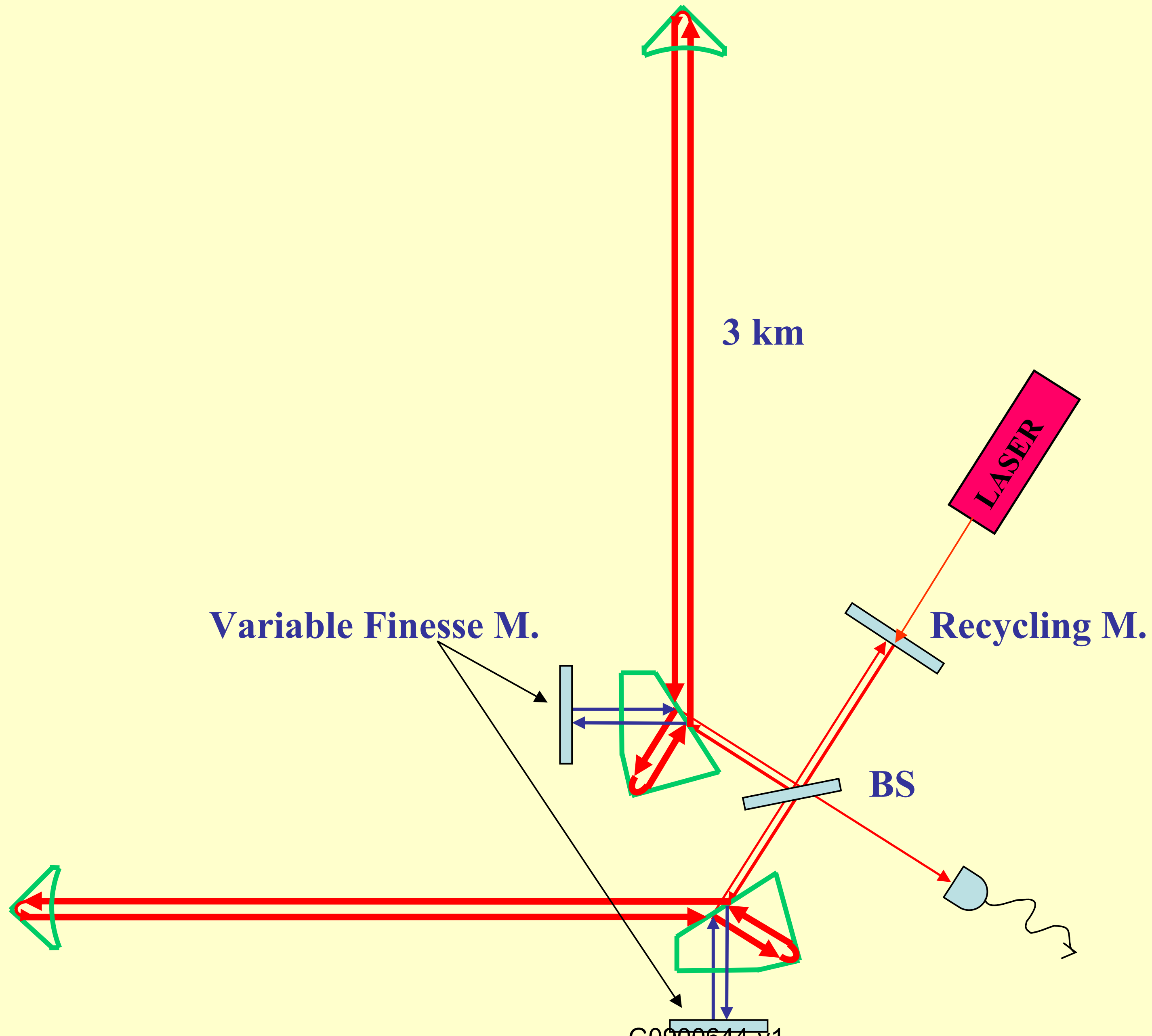
This equation shows that we may change the finesse by the factor $\Delta F = 1+\chi^2$ and still the phase shift due to thermal noises of M_3 in L_4 is depressed by the factor $\frac{R_1^2\chi^2}{1+\chi^2} \ll 1$ because $R_1^2 \ll 1$.

Then it seems that in this cavity system we may change finesse, and consequently storage time, without affecting :

1)Thermal noise from channel L_3-L_4, which for $\chi^2=0$ is zero.

2)Cavity geometry.

The independence of geometry from finesse seems to be a very important feature of this system.



G0900644-v1

Thermal Noise

With this optical configuration it is evident that every photon traverses a large amount of material; here the problem is transferred from normal mirror reflective coating bad loss angle to thermorefractive noise (TRN) of coatingless cavity. TRN spectral density is:

$$S_\phi(\omega) = \frac{16\pi L_{bulk} k_B T^2 \kappa}{\lambda^2 \rho^2 C^2 w_0^4 \omega} \beta^2$$

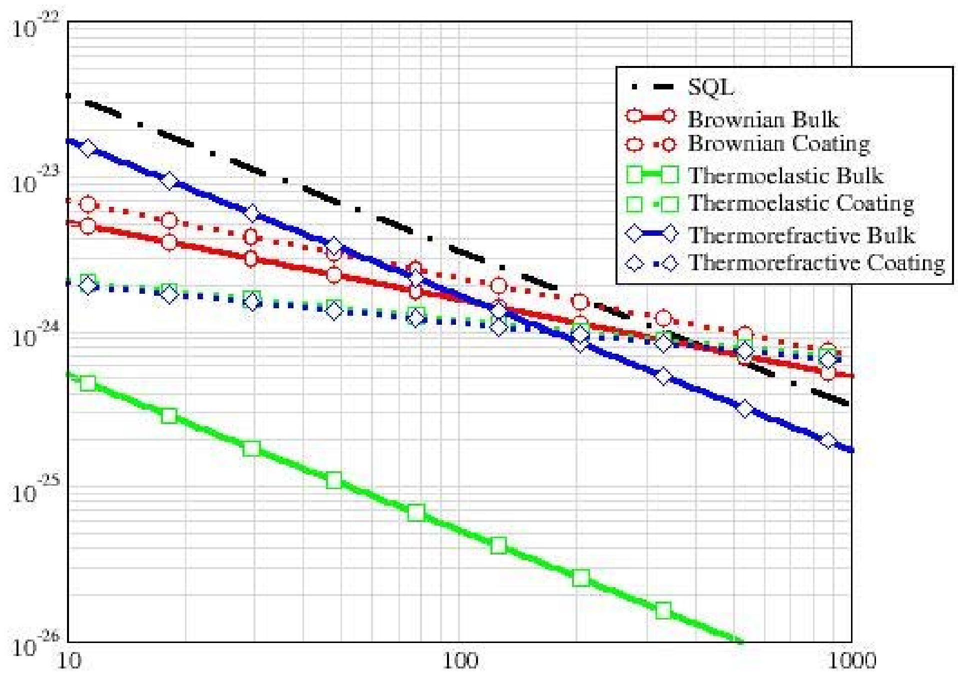
Where: L_{bulk} is the thickness of traversed material, w_0 the beam waist, ρ the bulk density, κ the bulk thermal conductivity coefficient, λ the wavelength, C the bulk thermal capacity and β is:

$$\beta = \frac{\partial n}{\partial T} = \frac{(n^2-1)(n^2+1)}{6n}\left[\frac{1}{\alpha_p}\frac{\partial \alpha_p}{\partial T} - 3\alpha\left(1 + \frac{\rho}{\alpha_p}\frac{\partial \alpha_p}{\partial \rho}\right)\right]$$

Where α_p is the bulk polarizability, n the bulk refractive index and α is the bulk linear thermal expansion coefficient.

We tried to find data on β at different temperature and we only found 300K data for Silica, which has large C and small κ.

Silica 300K



Data for Sapphire give bad results since C is small and κ is very large compared with Silica

Cristaline Quartz could be a good candidate for going to low temperature but we could not find any data.

G0900644-v1

Conclusions

This coatingless optical configuration has some interesting features:

1) **Since all the surfaces are optically matched, antireflective coatings are not needed.**

2) **Power can be injected in the cavity in a straightforward way; use of tunnel effect injection or transversal cavity are unecessary.**

3) **The optical scheme can be made with variable finesse. This property can also be applied to “normal” mirror configuration.**

It is evident that the draw back is the Thermo Refractive noise; an investigation for selecting crystaline bulks with low thermal conductivity and large thermal capacity is instrumental.