Bayesian Model Selection In Population-based Gravitational Wave Detection From Gamma-Ray Bursts

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Abstract

A fraction of short, hard gamma-ray bursts are believed to be associated with the coalescence of binary neutron star or neutron star-black hole systems, resulting in strong emission of gravitational radiation. The cosmological distance of these sources means, however, that the gravitational wave signal is likely to be very weak by the time it reaches the Earth. Several methods, such as the binomial test and the rank-sum test have been proposed that combine gravitational wave data from a sample of gamma-ray bursts to search for the statistical signature of multiple signals. Here, we introduce a new statistical test based on the Bayesian odds ratio, where we evaluate the posterior probability that the output from multiple matched-filter searches for inspiral signals are associated with a sample of gamma-ray bursts.

Gamma-ray Bursts

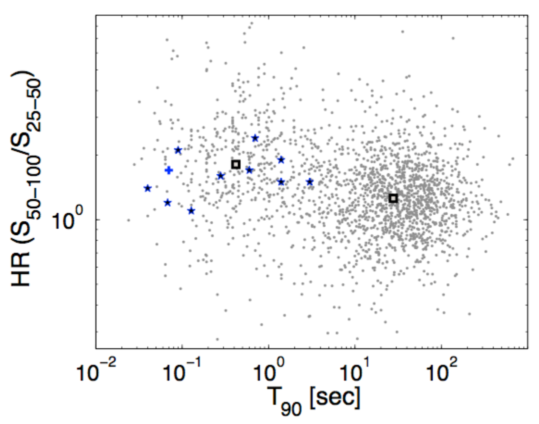


Image: sGRB hardness & duration (Credit: Ref. 1)

- Gamma ray bursts (GRBs) most luminous phenomena in universe
- Flash of gamma rays, followed by X-ray, UV, optical, radio afterglow
- Bimodal distribution of durations (T90 = time in which 90% of total emission occurs).

Characterise GRBs as:

- 'short' for T_{90} < 2s (harder spectra) - compact binary inspiral
- 'long' for T_{90} > 2s (softer spectra) - massive core collapse

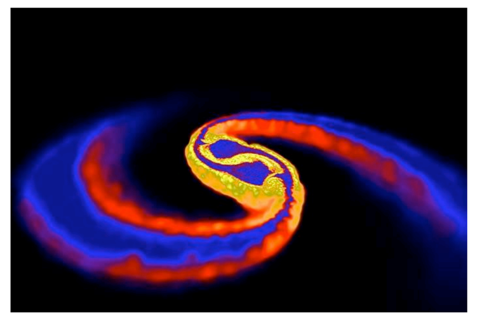


Image: Hydrodynamical star merger simulation (Credit: Daniel Price, Stephen Rosswog)

Gravitational Wave Emission From sGRBs

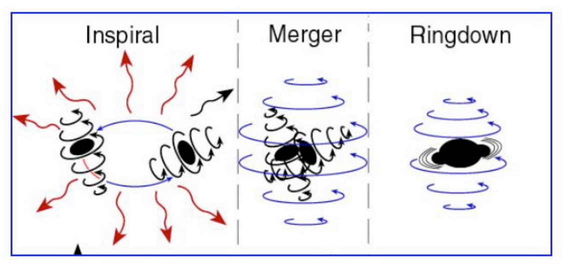


Image: Compact binary coalescence & GW emission (Credit: Kip Thorne)

- Compact binary system orbit decays:
- **inspiral:** chirp signal
- **merger**: messy burst of GWs
- **ringdown**: Resulting object (hyper-massive neutron star or black hole) undergoes post-merger oscillations, damped by GWs.

Inspiral Signal

- Gravitational wave strain in detector ⟶ $h(t) = F_+(\theta,\phi,\psi)h_+(t) + F_\times(\theta,\phi,\psi)h_\times(t)$
- F_+, F_x antenna response functions to GW polarisations (θ, ϕ - sky location angles, ψ - polarisation angle) ⟶

$$F_+ = -\tfrac{1}{2}(1+\cos^2\theta)\cos 2\phi\cos 2\psi - \cos\theta\sin 2\phi\sin 2\psi,$$
$$F_\times = \tfrac{1}{2}(1+\cos^2\theta)\cos 2\phi\sin 2\psi - \cos\theta\sin 2\phi\cos 2\psi.$$

- Post-Newtonian expression for inspiral (e.g., Refs. 6 and references therein) ⟶ $h(t) = \frac{1\mathrm{Mpc}}{D_{\mathrm{eff}}}[h_c(t)\cos\Phi + h_s(t)\sin\Phi],$
- Effective distance D_{eff} is function of distance r, inclination ι, F_+, F_x ⟶ $D_{\mathrm{eff}} = \frac{r}{\sqrt{F_+^2(1+\cos^2\iota)^2/4 + F_\times^2\cos^2\iota}}$

Searching For Gravitational Waves From GRBs

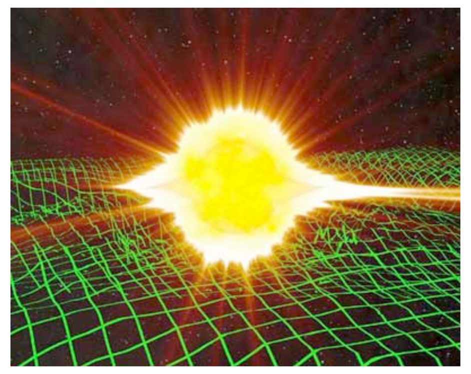


Image: BNS coalescence & gravitational field perturbation (Credit: John Rowe Animation, ATNF, CSIRO)

- See talk by **Nick Fotopolous** for recent **LIGO-Virgo** results for searching for **GRB inspirals**
- See talk by **Patrick Sutton** for recent **LIGO-Virgo** results for searching for **unmodelled GRB burst signals**
- Well-modeled inspiral signal from compact binary coalescence amenable to matched filter-based search
- Known progenitor, sky-location & event time constrain parameter space of search, boost sensitivity

Matched Filtering

Following Ref. 6:
- Single detector matched filter signal-to-noise ratio is defined

$$\rho^2 = \frac{(s|h_c)^2 + (s|h_s)^2}{\sigma^2}$$

- Where (a | b) is the inner product for detector's power spectrum S(| f |):

$$(a|b) := 4\,\mathrm{Re}\int_0^\infty \mathrm{d}f\,\frac{\tilde{a}(f)\tilde{b}^*(f)}{S(|f|)}$$

- Detector sensitivity to signal h_c characterised by:

$$\sigma^2 = (h_c|h_c)$$

- Signals present in detector output are characterised by:

$$\rho_h^2 := (h|h) = \frac{\sigma^2}{D_{\mathrm{eff}}^2}$$

- Construct 2-detector, 'combined SNR':

$$\rho = \sqrt{\rho_1^2 + \rho_2^2}$$

Population Based Searches

- Even with known waveform, sky location, time, GRB distances (<z> ~ 0.5) mean single signal is very difficult to detect.
- May still be multiple gravitational wave signals too weak to confidently detect individually.
- Distinction of 'on-source' and 'off-source' data makes it possible to search for a cumulative signature from multiple GRBs.
  - ➡ 'On-source' = data expected to contain putative gravitational wave signal.
  - ➡ 'Off-source' = data expected to only contain detector noise.
  - ➡ Common to test for consistency in distributions of on and off-source triggers.
  - ➡ First suggested for GW searches in Ref. 2, deployed in GW-GRB searches in Refs. 3-5. See also talks by **Fotopolous**, **Sutton**.
- Here, rather than test for consistency in trigger distributions, we look for excesses in threshold-crossing events....

Using Bayesian Model Selection

Bayes' theorem gives the posterior probability for a hypothesis H_i, given some cogent prior information I and a set of N mutually exclusive and exhaustive alternatives:

$$p(H_i|D,I) = \frac{p(H_i|I)p(D|H_i,I)}{\sum_{i=1}^{N} p(H_i|I)p(D|H_i,I)}$$

$p(H_i|I)$ ⟶ Prior belief in hypothesis H_i

$p(D|H_i,I)$ ⟶ Likelihood of data D, given hypothesis H_i

To construct our new detection statistic, assume combined **on-source GRB observations result in a population of N_{on} matched filter triggers**, where N_{on} is the number of loudest on-source trials with SNR crossing some initial detection threshold.

Similarly, let the combined **off-source trials result in N_{off} loudest SNRs** which cross some threshold. We then form the odds ratio $O_{\{s+b,b\}}$ (posterior probability ratio) between the following two models:

1. $\mathbf{M_{s+b}}$: the observations N_{on} are due to a foreground population (i.e., GRBs) with mean rate s and a background population with mean rate b.
2. $\mathbf{M_b}$: the observations N_{off} are due to the background population only

$$O_{\{s+b,b\}} = \frac{p(M_{s+b}|N_{\mathrm{on}},I)}{p(M_b|N_{\mathrm{on}},I)} = \frac{p(M_{s+b}|I)}{p(M_b|I)}\frac{p(N_{\mathrm{on}}|M_{s+b},I)}{p(N_{\mathrm{on}}|M_b,I)} = \pi_{\{s+b,b\}}B_{\{s+b,b\}}.$$

$$p(M_{s+b}|N_{\mathrm{on}},I) = \frac{1}{1+1/O_{\{s+b,b\}}}$$

Following Ref. 7, the 'Bayes' factor is:

$$B_{\{s+b,b\}} \approx \frac{N_{\mathrm{on}}!}{s_{\max}T_{\mathrm{on}}(N_{\mathrm{on}}+N_{\mathrm{off}})!}\sum_{i=0}^{N_{\mathrm{on}}}\frac{(N_{\mathrm{on}}+N_{\mathrm{off}}-i)!}{(N_{\mathrm{on}}-i)!}\left(1+\frac{T_{\mathrm{off}}}{T_{\mathrm{on}}}\right)^i$$

- T_{on} = on-source observation time
- T_{off} = off-source observation time
- s_{max} = maximum source rate

Synthesising A GRB-Triggered GW Search

Off-source

Following Ref. 6, the probability of obtaining a trigger in Gaussian noise with combined SNR ϱ^2 > loudest SNR, ϱ_*^2, is Poissonian:

$$P(\rho^2 > \rho_*^2) = \mathrm{e}^{-\rho_*^2/2}$$

with a mean rate:

$$\mu(\rho) = C\left(1+\rho^2/2-\rho_T^2\right)\mathrm{e}^{-\rho^2/2}$$

where ϱ_T^2 is the single detector SNR threshold and C is chosen such that the mean rate of the loudest events seen in background is 1.

★ Loudest events in the **off-source data are simulated** by **drawing N_{off} loudest values of ϱ^2 from a Poisson distribution where the expected loudest event in any given trial has ϱ=10** and the single detector SNR threshold is ϱ_T=5.5. Figure 1 demonstrates the result of simulating 10^5 loudest events from Gaussian noise.

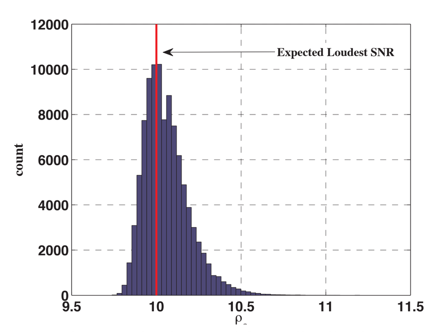


Figure 1: background SNR distribution

On-source

★ Loudest events in the **on-source data are simulated** by generating a population of GRBs uniformly distributed in volume up to 1 Gpc, isotropically distributed over the sky, with uniformly distributed polarisation between 0 and 2π and with inclination angle ι uniformly distributed such that | cos ι | > cos α, where α is the GRB jet opening angle.

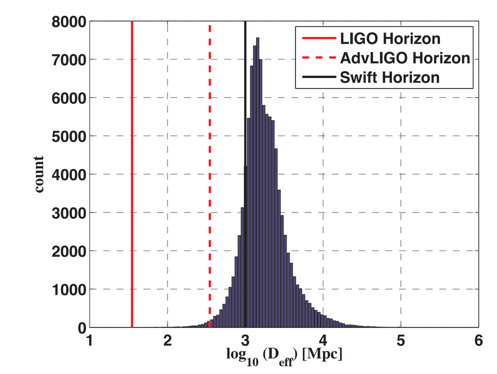


Figure 3: 10^5 simulated GRB Deff SNR distribution

SNR is then computed from D_{eff} and the horizon distance for which ϱ=8. If this is the loudest event in the on-source, we retain this trigger. Otherwise, a background trigger is used. Figure 3 demonstrates the effect of simulating the effective distances to 10^5 GRBs.

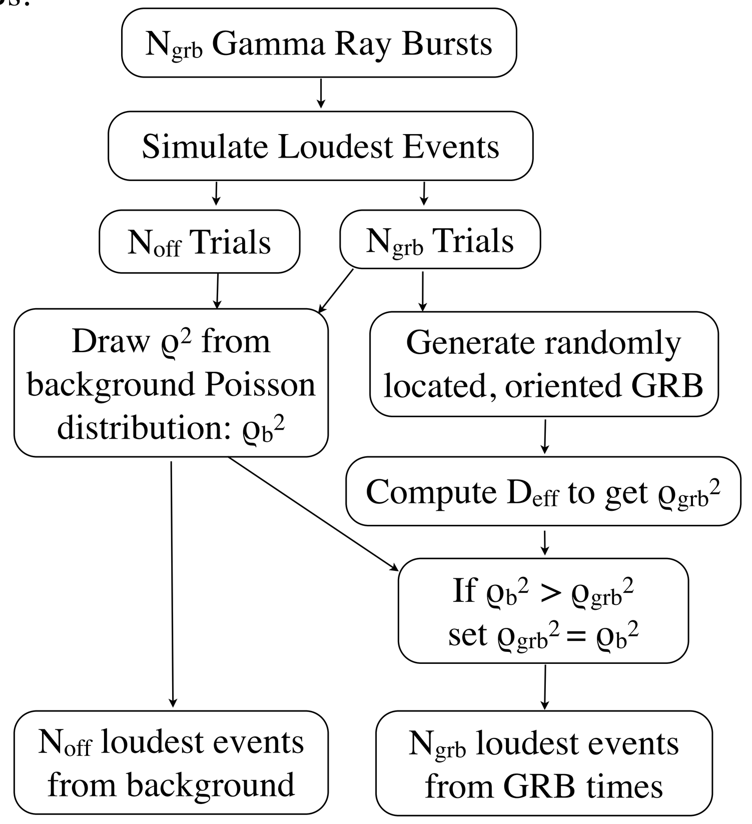
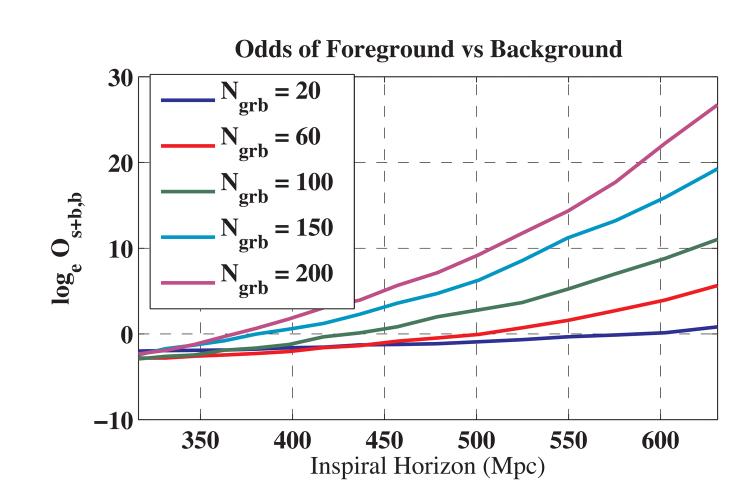
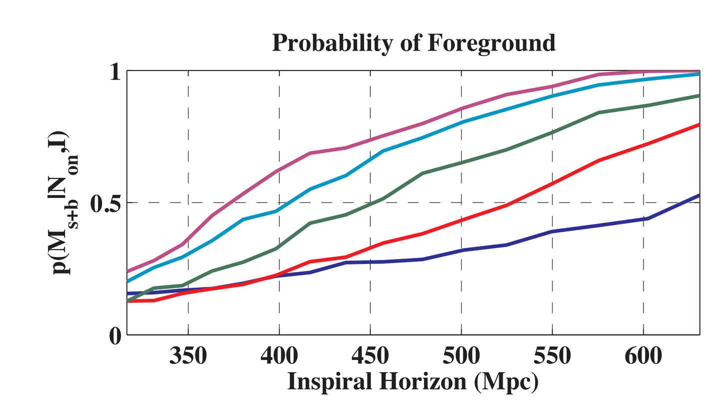


Figure 2: pipeline to synthesise a matched filter GRB-GW search

Results & Characterisation

- Simulate a GRB search by:
  - On-source time per GRB = 6s,
  - Noff = Number of off-source loudest triggers which cross detection threshold
  - Non = Number of on-source loudest triggers which cross detection threshold
  - Take detection threshold: combined SNR ϱ_c^2 > 10.8
  - Investigate variation of odds & probability of foreground signal with inspiral horizon for various GRB observations
  - Construct receiver operating characteristic curve for inspiral horizon & different numbers of GRBs:





Conclusions

- Search based on counting events unlikely to be more sensitive than individual loud triggers
- Interesting way to interpret searches for multiple GRBs
- Next steps:
  - get posterior on number of on-source events
  - compare with (e.g.,) rank-sum, binomial tests etc

References

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