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Stable Recycling Cavities for Advanced LIGO

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1 Summary

In February 2008 a stable optical geometry was adopted for the power- and signal-recycling cavities in Advanced LIGO. This note reviews the pros and cons of the stable and marginally stable designs, and motivates the choice of the former case.

2 *Stable* and *Marginally Stable*: what do they mean?

The stability condition for a two-mirror optical cavity is often expressed as: $0 \leq g_1 g_2 \leq 1$, where the cavity g-parameters are, $g_i = 1 - L/R_i$ (with cavity length L and mirror radii-of-curvature R_i). The g-parameters are related to the one-way Gouy phase shift by: $\Delta\psi = \arccos \pm\sqrt(g_1 g_2)$.

By *marginally stable* we mean a cavity that is close to the edge of the stability condition, and in particular for the recycling cavities, we're talking about a $g_1 g_2$ product close to unity. This is what we would have if we simply matched the recycling mirror ROC to the mode coming from the arm cavity. Viewed from the recycling cavity, the ITM looks like a mirror with ROC $= -1970\,\mathrm{m}/1.45 = -1359\,\mathrm{m}$. Propagating the beam at the ITM (with parameters $[\omega, R] = [5.55\,\mathrm{cm}, 1359.0\mathrm{m}]$), by $10\,\mathrm{m}$ (approximate RC length), gives a beam with an approximate ROC of $1369\,\mathrm{m}$. The g parameter product comes out to $g_1 g_2 = 1 - 1.2 \times 10^{-6}$, and the corresponding one-way Gouy phase is 0.0011 radian.[1] Clearly this is very close to the edge of stability. Such a cavity is also near-degenerate, so that higher-order spatial modes can build-up when the fundamental mode is resonant. The power recycling and signal recycling cavities (PRC and SRC) have finesses of approximately 100 and 25, respectively. This corresponds to a linewidth, in terms of one-way phase, of $0.03\,\mathrm{rad}$ (PRC) and $0.13\,\mathrm{rad}$ (SRC), both much larger than the higher-order mode spacing of $1.1\,\mathrm{mrad}$. Thus, about 30 (PRC) or 100 (SRC) higher-order modes would lie within the linewidth of the fundamental mode.

A stable cavity has a Gouy phase that is a significant fraction of π, and moves away from the near-degenerate condition so that higher-order modes are at least a linewidth away from the fundamental. In LIGO this can be done by essentially including the beam expansion/reduction telescopes inside the recycling cavities, as shown in Fig. 1. Both PRC and SRC beam paths are folded using the mode cleaner tube length, so that relatively low powered, spherical optics can be used with minimal off-axis distortion. In such a multi-element design, the stability condition and Gouy phase can be calculated using the ABCD matrix formalism.

3 Pros and Cons

[1] Initial LIGO's power recycling cavity has a Gouy phase of about $25\,\mathrm{mrad}$.

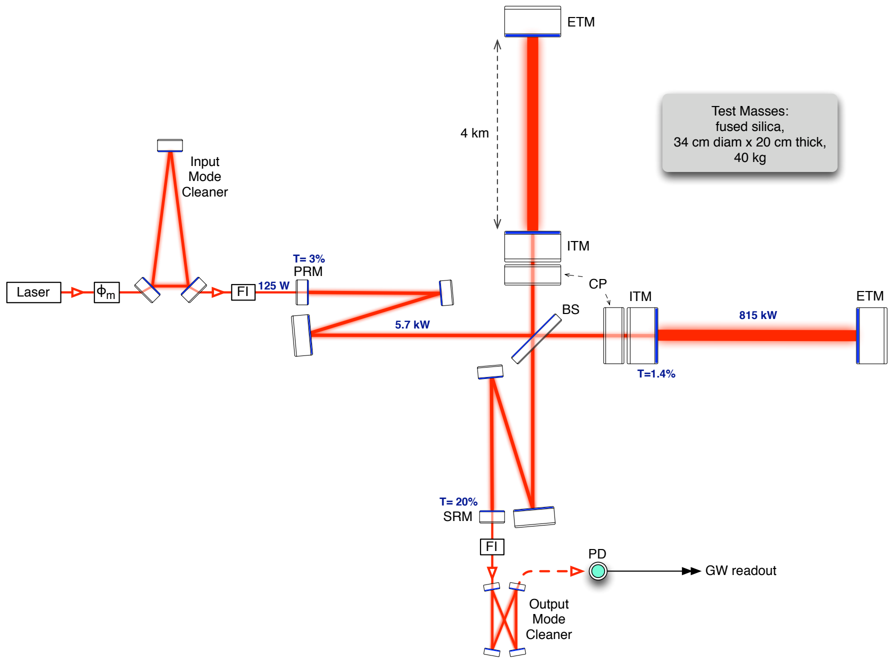

Figure 1: Interferometer layout with stable recycling cavities; both recycling cavity beam paths are folded within the mode cleaner tubes, between HAMs 2 and 3 for the PRC and between HAMs 4 and 5 for the SRC.

Pros	*Cons*
	Stable
Better GW sidebands	WFS signals reduced
Better RF sidebands	More triple suspensions
Easier to change SRM	More SRC path noise
POB beam easy to extract	Tighter tolerances on telescope ROC's
Adaptive mode-matching possible	More scattered light ports
	Marginally Stable
Simpler suspensions for telescope optics	GW sidebands more sensitive to distortions
Lower SRC path noise	RF sidebands more sensitive to distortions
	Need a large ITM wedge to dump ghost beams
	BS pick-off not available (very hard to get)
	Folded interferometer doesn't fit well

Table 1: Summary of pros and cons of stable and margninally stable recycling cavities.

3.1 Distortions in the recycling cavities

The main advantage of the stable cavity design is that it is less sensitive to optical distortions in the cavities. This has been studied by Yi Pan, using a numerical modal simulation.[2] He found that the loss in the signal-to-noise ratio (SNR) of the GW channel, due to curvature errors and higher order mode distortions in the input test masses and signal recycling mirror, could be much smaller for a stable recycling cavity, compared to the marginally stable case. As an example, with a common radius of curvature error in each input test mass of 5m, the SNR loss for the marginally stable case was calculated to be about 4%; in constrast, over a signficant range of Gouy phases in the stable regime, this loss is less than 0.5%. See his paper for the details.

The effect of distortions is also being studied with the FFT-based optical simulation (or SIS, Static Interferometer Simulation). The results of these simulations will be included in a future version of this note.

3.2 Suspensions

Since in the stable design the telescope mirrors are inside the recycling cavities, they require a lower noise level than in the marginally stable design. So in the stable design, all three mirrors in each recycling cavity would be mounted in a triple suspension (i.e., six triple suspensions total). One of these suspension pairs, the large triple suspension for the two mirrors (optically) closest to the beamsplitter, is common for the two recycling cavity designs. The additional triple suspensions for the stable case would be identical to the input mode cleaner suspensions, now known as small triple suspensions.

The telescope mirrors in the marginal design would be mounted in single-stage suspensions. For the large telescope mirror (MMT3) the initial LIGO LOS design would be modified to accommodate a larger optic; the other telescope mirrors would be mounted in initial LIGO small optic suspensions (SOS).

In summary, the stable design requires four additional small triple suspensions per interferometer; while they are more complex to assemble and install than a single-stage suspension, no new design work is needed since they would use the IMC suspension design. The marginally stable design would need some design work for the large-optic single-stage suspension.

3.3 Layout

A nice feature of the stable cavity design is that the ITMs and BS can have small wedge angles, and the first-order ghost beams from these optics can propagate back through the telescopes and be dumped when they are reduced in size. This avoids large, suspended beam dumps for full-sized ghost beams, and also significantly reduces cross-couplings associated with $\sim 1^\circ$ wedges.

The BS pick-off beam required for ISC sensing (or its equivalent) is much easier to extract

[2] "Optimal degeneracy for the signal-recycling cavity in advanced LIGO", Y. Pan, arXiv:gr-qc/0608128v1.

in the stable design. Here, we can simply use the/either leakage beam from PR2, where the beam is small. The marginally stable design would rely on a relatively large BS wedge to separate the BS AR beam; in practice this proved to be difficult to do within other layout constraints.

Finally, at this time no layout solution has been found that satisfies the cavity length constraints for the marginally-stable, folded interferometer case. This is not to say a solution could not be found, be it certainly proved to be difficult.

3.4 SRC path noise

A clear disadvantage of the stable geometry is increased optical path noise in the recycling cavities. This is more of an issue for the signal recycling cavity, since it has a higher coupling to the GW channel. Since two of the mirrors in the stable design are near-normal folding mirrors, their motion is doubled when calculating the path noise. Thus if all three mirrors in the SRC had the same displacement noise, the path noise for the stable geometry would be higher by a factor $\sqrt{(2^2 + 2^2 + 1)} = 3$.

Recently estimates have been made for the displacement noise in the triple suspensions, due to seismic and thermal noise. This is documented in T080192-01, "Displacement Noise in Advanced LIGO Triple Suspensions". This shows that the stable geometry is expected to meet its SRC path noise target (treated as a technical noise source) above about 30 Hz, but could exceed it at lower frequencies. Below 30 Hz, the dominant term in these estimates is vertical seismic platform noise, the there are several reasons to expect this term could be brought closer to the noise target: vertical ground noise is likely to be smaller than the level used in this estimate; vertical-to-horizontal coupling could potentially be smaller than the assumed level of 0.001; the triple suspensions can probably be made more compliant in the vertical direction to give more isolation.

3.5 Alignment sensing

Wavefront sensor alignment signals have been calculated for both geometries. For the marginally stable case, the one-way Gouy phase was 0.003 radian, while for the stable case the Gouy phase was 2.1 rad in the PRC and 0.51 rad in the SRC. Here is a summary of the differences in the sensing matrices:

- The PRM signal is (8 times) larger in the marginally stable cavity because the 10-mode of the RF sideband is (near-)resonant.
- The common-mode ITM (CITM) signal is (20 times) larger in the marginally stable cavity for the same reason. The PRM and CITM signals are nearly degenerate in the marginally stable case.
- The common-mode ETM signal is (2 times) smaller in the marginally stable case because the 10-mode of the carrier is close to antiresonant in the PR cavity.

- The SRM signal is (6 times) larger in the marginally stable cavity because the 10-mode of the higher-frequency RF sideband is close to resonant for this particular detuning. This increase in signal will disappear for other SR tunings (just a reminder).
- The differential-mode signal (in this port) does not change signicantly. The 10-mode of the lower frequency RF sideband is the main signal; it is not resonant in the SR cavity.
- The differential-mode ETM signal in the marginally recycling cavity increases because the 10- mode is resonant in the SR cavity for the RSE case (this will change if we detune the SR cavity).

In summary the marginally stable design gives larger alignment signals, but the stable design gives a more diagonal sensing matrix.

3.6 SRM flexibility

It is desirable to be able to relatively easily swap in signal recycling mirrors of different transmission. This should be possible with either design, as the SRM triple suspensions (large or small) are very similar in design, just different in scale. It is not clear if there would be any significant difference between the two designs in actually carrying out a swap (though of course smaller SRMs cost less). Looking into the future, implementing a variable transmission SRM sounds simpler with the small beam size of the stable cavity design.

4 Conclusion

Based on layout advantages and optical distortion tolerance, we opt for stable power- and signal-recycling cavities. This document is not intended to describe the specific design of these cavities; that is done within the Input Optics group, and will be detailed in the IO final design document. The final design is expected to use one-way Gouy phases of approximately 2 radians for the PRC and 0.5 radians for the SRC, as described in LIGO-P080004, *"Design of the Advanced LIGO Recycling Cavities"*.