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The Combined IFAR Statistic

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I. INTRODUCTION

The false alarm rate (FAR) is a statistic that can be defined for foreground signals when there is a measure of the background signal rate as well. For searches where the background rate changes for different trigger categories, it is useful to calculate the FAR with that category definition, which can then be recombined into a combined FAR (FARc). This statistic has been developed for the search for gravitational waves (GW) from low mass compact binary coalescences (CBC) during the first year of S5, what we will call the S5 1 Year Search. Section II discusses how different trigger categories were chosen for this search, Section III explains the FAR calculation, Section IV details how trigger categories are recombined, Section V presents comparisons of in-time triggers to the many different background trials, and Section VI describes implications for measuring the background probability for the loudest triggers.

II. DEFINITION OF TRIGGER CATEGORIES

In the S5 1 Year search there are several aspects that influence the background rate. These include the type of coincidence a trigger was found in (triple coincident triggers vs. double coincident triggers), the vetoes that were applied to the triggers, and the mass of the templates that correspond to the triggers. All of these aspects combined in order to define the trigger categories.

A. Coincidence Types

The first aspect we will focus on is the coincidence type. In the S5 1 Year search, we are searching for coincident triggers in time and two mass parameters between three GW detectors, the Hanford 4km interferometer (H1), the Hanford 2km interferometer (H2), and the Livingston 4km interferometer (L1). A trigger is deemed a double coincidence if it passes the coincidence requirements between two detectors. A trigger is promoted to a triple coincidence if there are three double coincident triggers that are all found in coincidence with each other. Since triple coincident triggers have an extra coincidence requirement, there is a much lower background rate for triple coincident triggers than for double coincident triggers.

Say each detector has a probability of producing a background trigger for a particular mass template at a particular time of ϵ_1. Adding the double coincident requirement then reduces that probability by an additional factor of ϵ_1, so that $\epsilon_2 = \epsilon_1\epsilon_1 = \epsilon_1^2$. Adding an additional coincident requirement added another factor of ϵ_1, so the probability of getting a triple coincidence trigger at a particular time is then $\epsilon_3 = \epsilon_1\epsilon_2 = \epsilon_1^3$.

This background rate reduction can be illustrated by looking at background H1H2L1 triggers and H1L1 triggers produced during triple coincident time (Fig. 1). For the S5 1 Year search the different trigger types we decided to use were H1H2L1, H1L1, and H2L1 triggers.

B. Veto Differences

Another aspect that affects the background rate is the differences in vetoes that are applied to different triggers. One set of vetoes which are trigger dependent are the amplitude vetoes applied between H1 and H2 triggers. These vetoes compare the effective distances found between coincident H1H2 triggers in H1H2L1 coincidences or, for H1L1 or H2L1 triggers, determine whether the other Hanford detector should have also produced a trigger at that time. These vetoes can only be applied both Hanford detectors were in operation resulting in background rate differences between H1L1 triggers from triple coincident time versus H1L1 trigger from double coincident time, and similarly for H2L1 triggers. This effect is shown for H2L1 trigger in Fig. 2, which clearly shows broader background distributions for H2L1 time where the veto in not applied even though there is an order of magnitude for triple coincident time. This effect is also stronger for H2L1 triggers in triple coincident time than for H1L1 triggers in triple coincident time

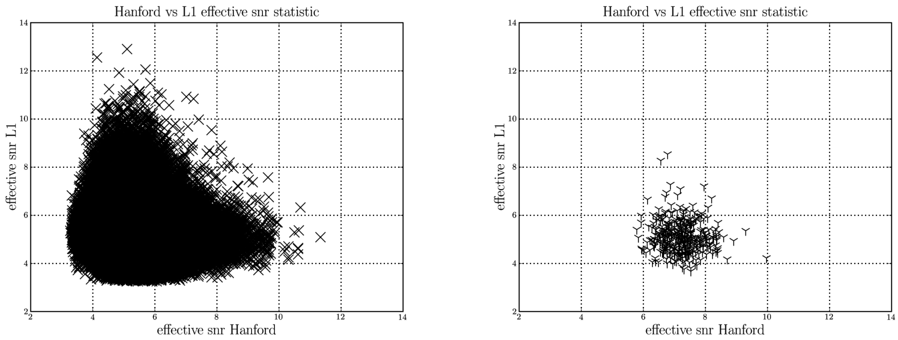

FIG. 1: In this figure we show the effective SNR for background H1L1 double coincident triggers during triple coincident time (left) and H1H2L1 triple coincident triggers during triple coincident time (right). The additional coincidence requirement for H1H2L1 triggers is seen to drastically reduce the background trigger rate.

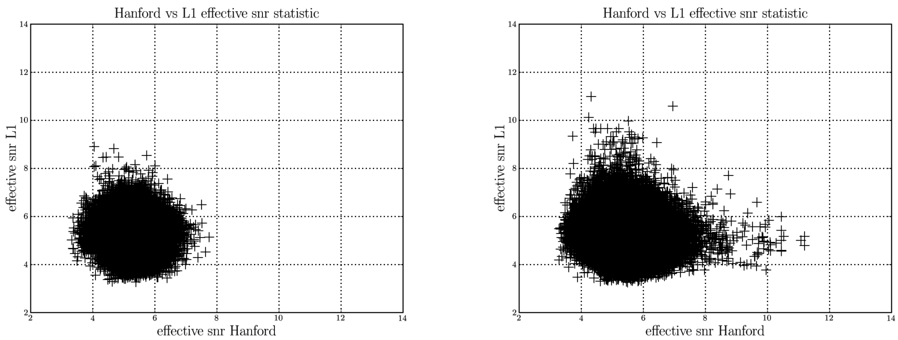

FIG. 2: In this figure we show the effective SNR for background H2L1 double coincident triggers during triple coincident time (left) and H2L1 double coincident triggers during double coincident time (right). The Hanford amplitude veto drastically reduces the brackground rate during triple coincident times, even though there is an order of magnitude more triple coincident time than double coincident time.

(Fig. 3). This is due to the fact that the H2 detector is roughly half as sensitive as the H1 detector, which means that if there is a trigger in H2, most of the time H1 should also have produced a trigger at that time.

C. Mass Differences

The S5 1 Year search is search over a much larger parameter space than a single search has covered before. Specifically, the mass space we cover with templates goes from a minimum total mass $M = m_1 + m_2$ of $2M_\odot$to a maximum total mass of $35M_\odot$with a minimum component mass of $1M_\odot$. In order to cover this large space, we use around 6000 templates whose time durations vary from over 44 seconds to under 0.34 seconds.

Due to the large variation in the length of the templates, we find that there is a larger variation in the templates' responses to noise glitches in the detectors where high mass templates tend to pick up the noise glitches as triggers with higher effective SNR. This can be seen in Fig. 4 where we plot the cumulative distributions of background H1L1 double coincident triggers during triple coincident time versus effective SNR separated by small bands in the mean chirp mass $M_c = M\eta^{3/5}$, where η is the symmetric mass ratio $\eta = m_1 m_2/M^2$. Three populations are found with different slopes for the distributions signifying different background rates. The chirp mass divisions for these different populations occur at $M_c/(.25)^{3/5} = 8M_\odot$ and $M_c/(.25)^{3/5} = 17M_\odot$.

We divide the trigger of a given type into these three different populations and find that indeed the higher chirp mass templates have a broader effective SNR distribution for background triggers signifying a different background rates for the different chirp mass populations (Fig. 5).

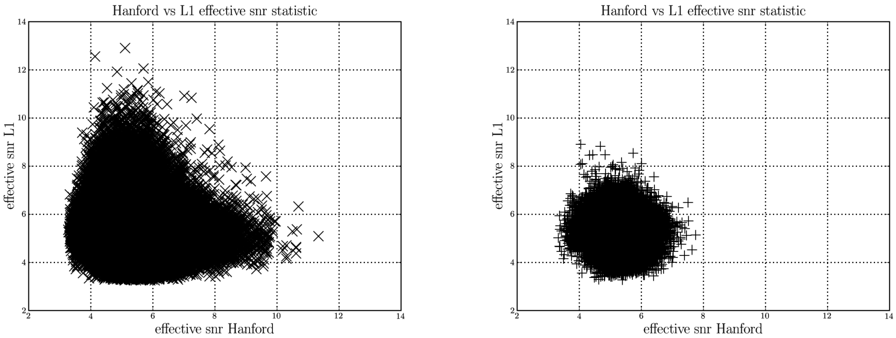

FIG. 3: In this figure we show the effective SNR for background H1L1 double coincident triggers during triple coincident time (left) and H2L1 double coincident triggers during triple coincident time (right). The Hanford amplitude veto is seen to be stronger for H2L1 triggers than for H1L1 triggers because the H2 detector is roughly half as sensitive as the H1 detector, which means that if there is a trigger in H2, most of the time H1 should also have produced a trigger at that time.

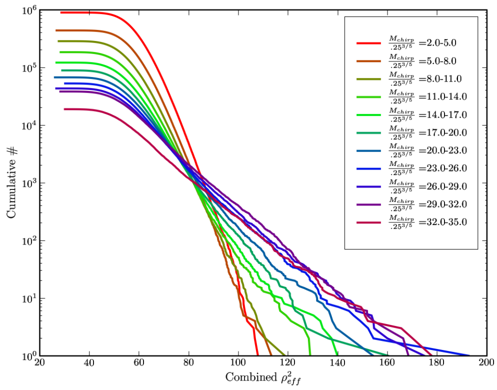

FIG. 4: In this figure we show the cumulative distributions of background H1L1 double coincident triggers during triple coincident time versus effective SNR separated by small bands in the mean chirp mass for the coincidnece. Three populations are found with different slopes for the distributions signifying different background rates. Similar results are obtained by looking at plots for different coincidence types.

D. Trigger Categories

Due to all of the above effects, we end up dividing the triggers from the S5 1 Year search into 15 different categories during our three different observation times. During triple coincident times we have 3 coincidence types subdivided into 3 chirp mass bins, resulting in 9 trigger categories. During each of the double coincident times we have 1 coincidence type subdivided into 3 chirp mass bins, resulting in 3 trigger categories for each double coincident time.

III. FAR: FALSE ALARM RATE

The FAR can be calculated using any intermediate statistic ranking the “loudness” of triggers (such as the effective SNR for the S5 1 Year search) where a higher intermediate statistic corresponds to a larger excursion for background behaviour. The FAR for any trigger is calculated using:

$$\mathrm{FAR} = \frac{\sum\limits_i n_i}{\sum\limits_i T_i} \tag{1}$$

where n_i is the number of background triggers in background trial i with intermediate statistic greater than or equal to the one in question, and T_i is the amount of analyzed time in background run i.

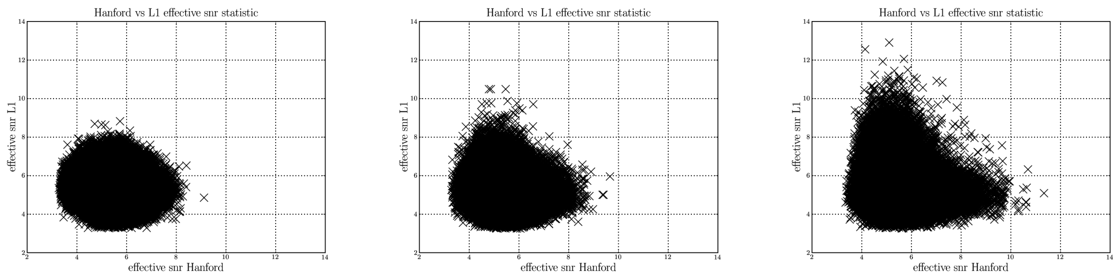

FIG. 5: In this figure we show the effective SNR for background H1L1 double coincident triggers during triple coincident time for the low chirp mass bin (left), the medium chirp mass bin (center), and the high chirp mass bin (right). Higher chirp mass templates have a shorter time duration, which leads to a larger response noise glitches, thus a broader effective SNR distribution for background triggers.

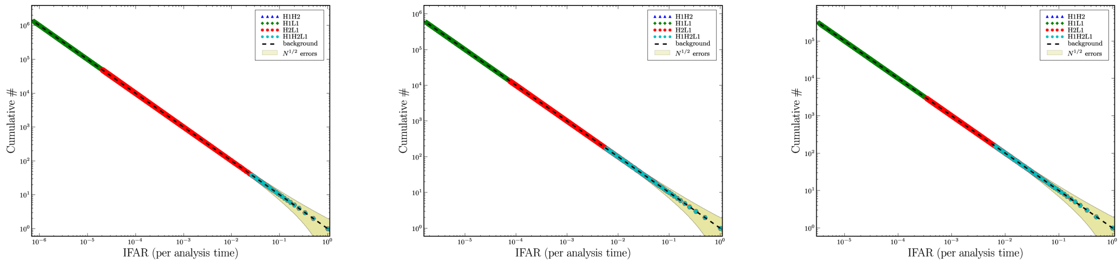

FIG. 6: In this figure we show the cumulative number vs. the IFAR divided by the analysis time for nine categories of background triggers across three plots. Because they are background triggers, they should, and do, fall exactly on the background line.

For individual trigger categories, the maximum FAR is found by:

$$\mathrm{FAR}_{\max,j} = \frac{N_j}{\sum\limits_i T_i} \tag{2}$$

where N_j is the total number of background triggers in the j^{th} trigger category. This can be converted into a minimum inverse false alarm rate (IFAR) trivially by taking the inverse.

The expected number of triggers below a particular FAR due to background is $\mathrm{FAR} \times T_0$ where T_0 is the foreground time analyzed. Another way to say this is the expected number of triggers above a particular IFAR due to background is T_0/IFAR where T_0 is the foreground time analyzed, as seen in Fig. 6.

IV. FARC: COMBINED FALSE ALARM RATE

When combining categories, we think of the different categories as different trials. If there are m trials, then we expect that of those trials there will be m triggers with an IFAR $\geq T_0$. A simple way to normalize the FAR to bring the expected number of triggers with an IFAR of T_0 back to one is:

$$\mathrm{FAR}' = m \times \mathrm{FAR}. \tag{3}$$

This can be seen in Fig. 7.

The simple way of combining shows that, moving from right to the left, there is a kink in the background triggers for whenever you reach a minimum IFAR for a trigger category. The reason these kinks occur is because once we reach a minimum IFAR for a trigger category, there is then one less "trial" for us to combine. A better thing to do at that point is to normalize the FAR by the remaining number of categories rather than the total. In analytic form, the FARc becomes:

$$\mathrm{FARc} = \left[\sum_{j=1}^{p} \Theta\left(\mathrm{FAR}_{\max,j} - \mathrm{FAR}\right)\right] \times \mathrm{FAR} + \sum_{j=1}^{p} \left[\Theta\left(\mathrm{FAR} - \mathrm{FAR}_{\max,j}\right) \mathrm{FAR}_{\max,j}\right] \tag{4}$$

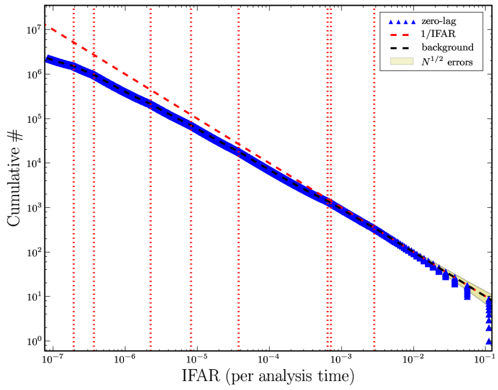

FIG. 7: In this figure we show the cumulative number vs. the IFAR divided by the analysis time for nine categories of background triggers combined by in the simple way. Because they are background triggers, they should, and do, fall exactly on the background line. The vertical, red, dotted lines denote the minimum IFAR for the different trigger categories.

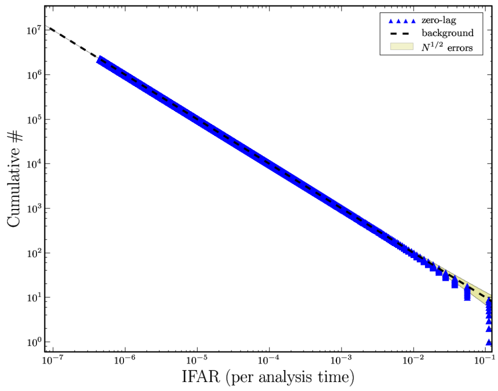

FIG. 8: In this figure we show the cumulative number vs. the IFARc divided by the analysis time for nine categories of background triggers. Because they are background triggers, they should, and do, fall exactly on the background line.

where p is the number of categories of triggers and $\Theta(x)$ is the Heaviside function. Calculating the FARc in this way can be seen in Fig. 8.

V. PLOTTING BACKGROUND TRIALS: LIGHTNING BOLT PLOTS

In addition to plotting the IFARc of foreground triggers, we can also plot the IFARc of each of the individual background trials (*i.e.* time slides). In this case, the IFARc has been normalized by:

$$\mathrm{IFARc}_i = \mathrm{IFARc} \times \frac{T_0}{T_i} \tag{5}$$

where i corresponds the different background runs. An example plot of this is shown in Fig. 9.

VI. BACKGROUND PROBABILITY

Using the FARc as the detection statistic allows a very easy calculation of the background probability P_b [1, 2]. Assuming a poisson distribution, the probability of getting zero background triggers louder than the loudest zero-lag trigger (*i.e.* with a FARc lower than the FARc of the loudest zero-lag trigger since low FARc's are more significant)

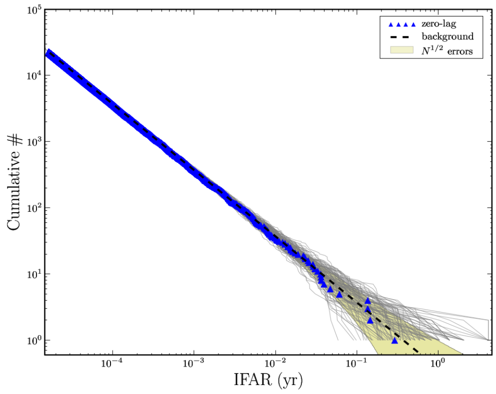

FIG. 9: In this figure we show the cumulative number vs. the IFARc for nine categories of foreground triggers from H1H2L1 observation time in the S5 First Year Low Mass Compact Binary Coalescences Search.

is given by:

$$P_b = e^x \tag{6}$$

where $x = -\mathrm{FAR} \times T$, FAR is the FARc of the loudest zero-lag trigger, and T is the total analyzed time searching for zero-lag triggers.

[1] R. Biswas, P. R. Brady, J. D. E. Creighton, and S. Fairhurst (2007), arXiv:0710.0465 [gr-qc].
[2] P. R. Brady and S. Fairhurst, Class. Quant. Grav. **25**, 1050002 (2008), arXiv:0707.2410 [gr-qc].