Design of Beamsplitter Suspension for Advanced LIGO.

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1. Introduction.

The beamsplitter (BS) is suspended in a BSC chamber, and as such it is provisionally assumed to have a quadruple pendulum suspension similar to the ETM and ITM. There is space to have a four stage suspension. However the noise requirements for the BS are relaxed from those of the TMs and thus it is worthwhile exploring whether a triple suspension could meet the design requirements. A triple suspension has several obvious advantages over a quad. Firstly it would be a less complex suspension to design and build – with likely savings in effort, time and money. Secondly we have already built up a lot of experience with triple pendulum designs, both from GEO and from the modecleaner and recycling mirror designs developed for Advanced LIGO.
In this document we present a preliminary triple pendulum design which appears to meet the requirements.

2. Beamsplitter Requirements.

The beamsplitter (BS) will be fused silica right circular cylinder, 35 cm diameter and 6 cm thick, giving a mass of 12.7 kg. Currently it is expected to have a wedge angle of 1.3^{o}.

The noise requirements are currently under review while the optical layout and orientation of wedge angles are being considered. Assuming at present that a vertical orientation of wedge is used, the relevant vertical – length coupling factor with respect to the differential change in length of the Michelson arms due to this wedge has been computed to be 1.04e-3 at the LHO site and smaller at LLO (ref D Coyne, e-mail 27 Jan 04 and document T040007, currently being revised). Taking into account motion along the BS optic axis (x) and vertical (z) direction, the overall requirement on displacement noise at 10 Hz from all sources can be written as

sqrt(2*x^2 + (1.04e-3*z)^2) < sqrt(2)*2e-17 m/rt Hz (1)

(ref P Fritschel, e-mail 27 Jan 2004)

Note that 2e-17 m/rt Hz is the value which is quoted in the design requirements document (T010007-01 – currently being updated) for the requirement on BS horizontal displacement noise along its optic axis (i.e. at 45 degrees to the Michelson arms). This value is therefore multiplied by sqrt(2) in the above relationship to give the equivalent number in terms of differential change in length of the Michelson arms. For the purposes of considering the motion of the beamsplitter itself in its x and z directions it is instructive to rewrite (1) as sqrt(x^2 + [(1.04e-3)/sqrt2)*z]^2) < 2e-17 m/rt Hz (2)

This is directly comparable to how the requirements have been written before. With no z coupling the requirement on the x (optics axis) motion of the beamsplitter is 2x 10^{-17} m/√ Hz at 10 Hz. Including the z motion, we see that the "effective" vertical-horizontal coupling appears to be smaller by √2 than the vertical-length coupling number quoted above, giving a value of 7.4 x 10^{-4}. Now in general we have been taking as a conservative value a cross-coupling of 10^{-3} arising from mechanical offsets in the suspension design, and so for the purposes of this document I will consider that we require to satisfy

sqrt(x^2 + [1e-3*z]^2) < 2e-17 m/rt Hz (3)

3. Choice of Parameters.

The working design which has been investigated is of a triple suspension with approximately equal masses (~12.7 kg each) and equal wire lengths of 60 cm at each stage. The choice of equal masses and equal wire lengths as a baseline has come from experience with previous designs and leads to good coupling of modes. In addition using three equal lengths gives the best isolation for a given overall length. As a start we have taken other parameters from the current recycling mirror triple suspension (which is similar in mass), and modified it where necessary to take into account the significantly different aspect ratio of the optic. The overall length was chosen to satisfy the available length for a beamsplitter suspension in a BSC (noting that this is 70 mm longer than for an ETM) *prior* to the recent considerations to reduce overall length of BSC suspension structures summarized in "Investigation of Wire Lengths in Advanced LIGO Quadruple Pendulum Design for ETM/ITM" NAR , 26 Jan 2004 (DCC # xxx) and thus will need revisited (see comments in conclusions). The final stage of the suspension consists of 4 silica fibres of circular cross-section, 140 micron radius (stress ~500 MPa). This gives a highest vertical frequency of 10.9 Hz and a first violin mode at 400 Hz , which satisfies the requirements expressed in the document on low-frequency cut-off (P Fritschel et al T020034-00-D), which requires a vertical mode of 12 Hz or lower, and first violin mode 400 Hz or higher. A full parameter set is given in Appendix A.

4. Suspension Thermal Noise.

In Figure 1 the suspension thermal noise estimate for a beamsplitter suspended as a triple pendulum is shown. The data was computed by G Cagnoli using his MAPLE code (corresponding to the code in BENCH), modified by NAR to reflect the new value of coupling factor, taken as 0.001. (For reference, the coupling factor quoted in T010007-01 was 0.014). Two curves are shown. The black curve is the x motion assuming zero coupling. The red curve is sqrt(x^2 + (1e-3*z)^2).

We see that the x noise alone lies well below the requirement at 10 Hz. The overall noise is dominated by the z contribution up to around 20 Hz. At 10 Hz the overall noise is below the requirement by a factor of 10, at a value of 2.0 x 10^{-18} m/√ Hz, with a narrow spike of higher noise around 11 Hz at the vertical resonance.

The choice of 140 micron radius was conservative. By decreasing the radius to 113 micron, which takes the stress to ~770 MPa, comparable to that proposed for the ETM and ITM suspensions, the highest vertical mode moves down to 8.9 Hz (see figure 3).

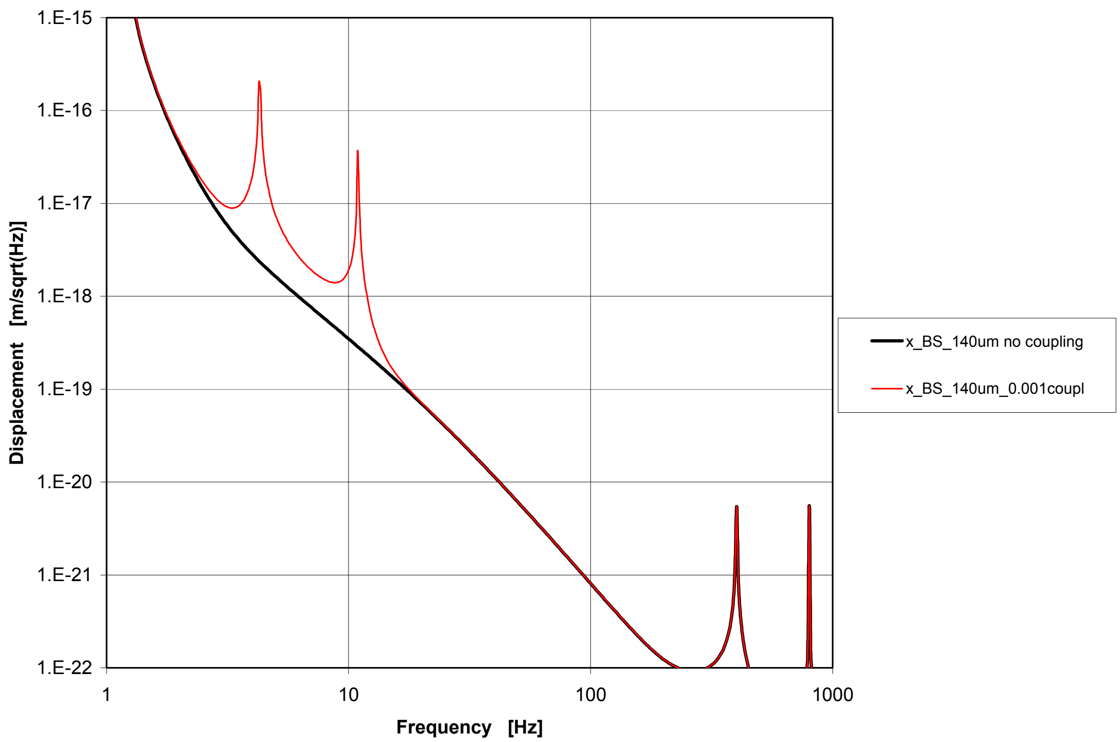

Figure 1. Suspension thermal noise for beamsplitter. Black curve – horizontal (x) motion assuming no coupling from vertical. Red curve – total horizontal motion including coupling from vertical, as given in the text.

5. Seismic Isolation, Mode Frequencies and Damping

The longitudinal and vertical transfer functions are given in figures 2 and 3. The longitudinal transfer function (with damping time ~8 secs) has a magnitude at 10 Hz = 1.3×10^{-6}. This, combined with active platform noise level of 2×10^{-13} m/√ Hz at 10 Hz, gives a noise level at the optic of 2.6×10^{-19} m/√ Hz at 10 Hz. The vertical transfer function = 2.3×10^{-2} at 10 Hz. (with damping time 4 secs), giving vertical noise level at optic of 4.6×10^{-15} m/√ Hz. Including a 10^{-3} coupling factor gives a residual noise level in the horizontal of 4.6×10^{-18} m/√ Hz. Taking the quadratic sum of these numbers yields a total essentially the same as the noise due to vertical alone, and lying well below the requirement.

For completeness the TF for thinner fibres of radius 113 micron is also included – showing the vertical peak at 8.9 Hz and similar performance at 10 Hz.

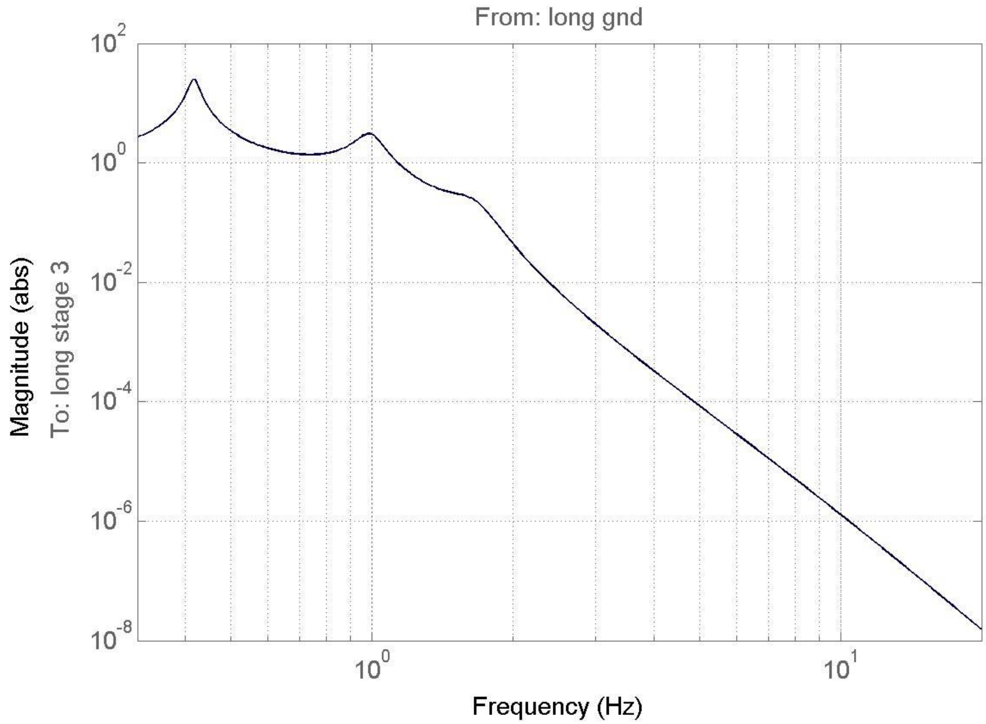

Figure 2. Longitudinal transfer function for beamsplitter triple suspension.

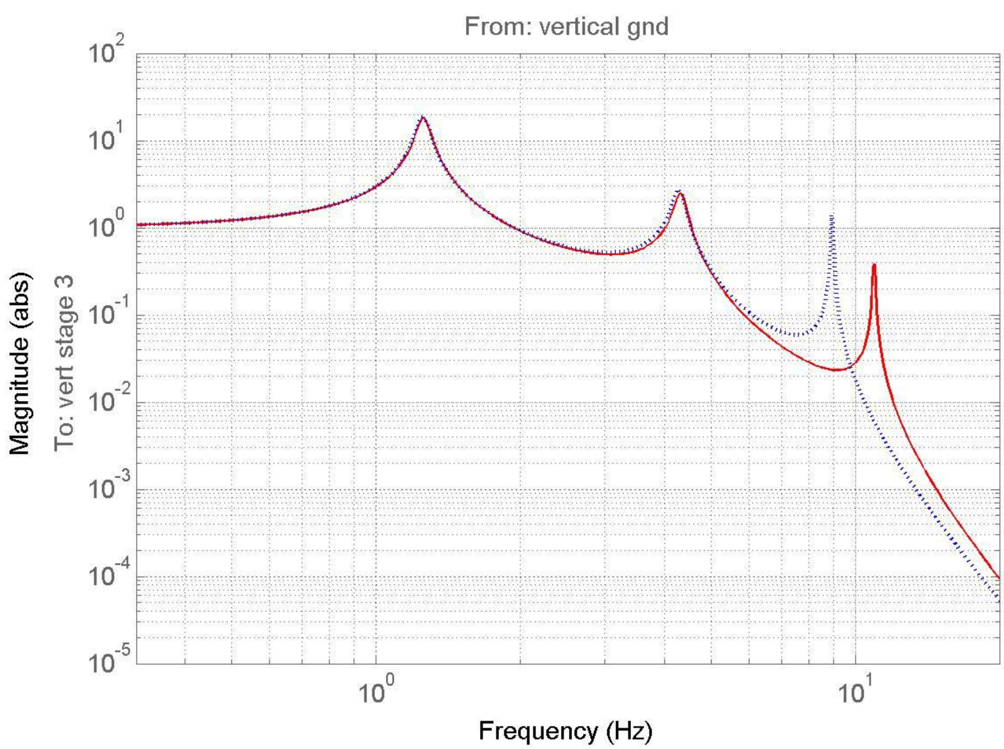

Figure 3. Vertical transfer function for beamsplitter triple suspension. The red curve corresponds to radius of silica fibres of 140 micron and the blue (dotted) curve to radius of 113 micron.

The mode frequencies for the preliminary design are given at the end of Appendix A. They are in general similar to other triple pendulum designs. Of particular note is that the lowest pitch mode is at 0.3 Hz, somewhat less than usual, which is a result of the more extreme aspect ratio. However that mode and all the other low frequency modes are well coupled and adequately damped using forces applied to the top mass in the usual way.

6. Overall Noise

We can combine quadratically the suspension thermal and residual seismic noise from horizontal and vertical at 10 Hz (dominated in both cases by the vertical contribution) yielding a total of $(2^2 + 4.6^2)^{1/2} \times 10^{-18}$ m/√ Hz = 5×10^{-18} m/√ Hz, well under the overall requirement of 2×10^{-17} m/√ Hz. Using the MATLAB model we can also estimate the magnitude of pitch and yaw contributions. The larger of these TFs at 10 Hz is for yaw, at 5.1×10^{-6}. An angular input at the platform of around 4×10^{-10} rad/√Hz and a 1mm beam offset would give a horizontal noise level of 2×10^{-18} m/rt Hz at 10 Hz (10 times smaller than the overall requirement) Such an input is well above the expected residual angular motion of the platform.
A further consideration is that of noise introduced by local control. Work is being carried out to develop low noise sensors which combined with steep electronic filtering and some relaxation of requirement on damping time in science operation for some modes should yield a workable solution. However regardless of this development, eddy current damping is a practical fallback to active damping for this suspension, for all degrees of freedom of the optic.

7. Conclusions.

We have examined a possible triple pendulum suspension for the beamsplitter and conclude that it appears to satisfy the noise requirements. We stress that the design shown here is preliminary and the parameter list is only given for guidance. For example in the light of the discussion referred to in section 3 on overall length of structure for installation in a BSC chamber, it is likely we would wish to reduce the suspension length by around 10 cm. We have taken a brief look at a design with the upper two stages each reduced by 5 cm and seen that this will have very little effect on the performance (vertical noise, which dominates, is unaffected by shortening the wire lengths). Other issues which are still open include the radius of the final stage silica fibres – where we can put the highest vertical mode below 10 Hz by using a stress level the same as that proposed for the E and ITMS, or use a slightly more conservative design and still satisfy requirements.

Further work is needed to flesh out the design in such areas as blade parameters and detailed design of uppermost mass. In addition details of global control requirements have to be worked out for the beamsplitter, so that the question of the design of a reaction chain (or indeed the need for a reaction chain) can be addressed. However none of these issues should materially affect the results given here, and hence our overall conclusion is that a triple pendulum suspension appears to meet all the requirements for the beamsplitter suspension. We thus propose that a triple pendulum suspension design is adopted for the beamsplitter suspension.

Appendix A .

A.1 Summary of parameters used in the MAPLE code for calculation of pendulum thermal noise.

```
> m[1] := 12.6 ;# [kg]
> m[2] := 12.7 ;# [kg]
> m[3] := 12.7 ;# [kg]
> L[1] := 0.598 ;# [m]
> L[2] := 0.588 ;# [m]
> L[3] := 0.600 ;# [m]
>
> N[1] := 2 ;
> N[2] := 4 ;
> N[3] := 4 ;
>
> r[1] := 300e-6 ;# [m]
> r[2] := 200e-6 ;# [m]
> r[3] := 140e-6 ;# [m]
>
> t[1] := 2400e-6 ;# [m]  Thickness of the fist blades
> t[2] := 1600e-6 ;# [m]  Thickness of the fist blades
>
> f_vert[1] := 2.591 ;# [Hz]  Uncoupled vertical frequency 1st stage
> f_vert[2] := 3.0432 ;# [Hz]  Uncoupled vertical frequency 2nd stage
>
> Ya[Maraging] := 186e9 ;# [Pa]
> Ya[Steel] := 165e9 ;# [Pa]
> Ya[Silica] := 70e9 ;# [Pa]
> Ya[Sapphire] := 400e9 ;# [Pa]
>
> phi[Maraging] := 1e-4 ;
> phi[Steel] := 2e-4 ;
> phi[Sapphire] := 5e-9 ;
> phi[Silica] := 3e-11 ;# Surface losses have to be added
>
> K[Maraging] := 20 ;# [W/m/K]
> K[Steel] := 49 ;# [W/m/K]
> K[Silica] := 1.38 ;# [W/m/K]
> K[Sapphire] := 33 ;# [W/m/K]
>
> C[Maraging] := 460 ;# [J/kg/K]
> C[Steel] := 486 ;# [J/kg/K]
> C[Silica] := 772 ;# [J/kg/K]
> C[Sapphire] := 770 ;# [J/kg/K]
>
```

```
> alpha[Maraging] := 11e-6 ;# [1/K]
> alpha[Steel] := 12e-6 ;# [1/K]
> alpha[Silica] := 5.1e-7 ;# [1/K]
> alpha[Sapphire] := 5.1e-6 ;# [1/K]
>
> beta[Maraging] := -2.5e-4 ;# [1/K]
> beta[Steel] := -2.5e-4 ;# [1/K]
> beta[Silica] := 2e-4 ;# [1/K]
> beta[Sapphire] := 0e-6 ;# [1/K]
>
> rho[Maraging] := 7800 ;# [kg/m^3]
> rho[Steel] := 7800 ;# [kg/m^3]
> rho[Silica] := 2200 ;# [kg/m^3]
> rho[Sapphire] := 3980 ;# [kg/m^3]
>
> sigma[Maraging] := 0.3 ;
> sigma[Steel] := 0.3 ;
> sigma[Silica] := 0.17 ;
> sigma[Sapphire] := 0.23 ;
>
> k[B] := 1.38e-23 ;# [J/K]
> Temp := 290 ;# [K]
> g := 9.81 ;# [m/s^2]
```

A.2 Summary of parameters used in the MATLAB model for calculation of transfer functions, and frequencies of modes.

```
        m1: 1.2600e+001
 material1: 'steel'
       I1x: 1.5225e-001
       I1y: 2.7405e-002
       I1z: 1.3240e-001
        m2: 1.2711e+001
 material2: 'silica'
        ix: 6.0000e-002
        ir: 1.7500e-001
       I2x: 1.9464e-001
       I2y: 1.0114e-001
       I2z: 1.0114e-001
        m3: 1.2711e+001
        tx: 6.0000e-002
        tr: 1.7500e-001
       I3x: 1.9464e-001
       I3y: 1.0114e-001
       I3z: 1.0114e-001
        l1: 6.0000e-001
```

l2: 6.0000e-001
l3: 6.0000e-001
nw1: 2
nw2: 4
nw3: 4
r1: 3.0000e-004
r2: 2.0000e-004
r3: 1.4000e-004 (* alternate value 1.13e-4)
Y1: 2.2000e+011
Y2: 2.2000e+011
Y3: 7.0000e+010
l1b: 2.5000e-001
a1b: 6.2500e-002
h1b: 2.4000e-003
ufc1: 2.5910e+000
st1: 7.7709e+008
intmode_1: 1.4457e+002
l2b: 1.4000e-001
a2b: 2.5778e-002
h2b: 1.6000e-003
ufc2: 3.0432e+000
st2: 7.9365e+008
intmode_2: 3.0733e+002
d0: 1.0000e-003
d1: 1.0000e-003
d2: 1.0000e-003
d3: 1.0000e-003
d4: 1.0000e-003
su: 0
si: 1.5000e-002
sl: 5.0000e-003
n0: 7.7000e-002
n1: 1.3000e-001
n2: 6.0000e-002
n3: 1.8150e-001
n4: 1.7650e-001
n5: 1.7650e-001
tl1: 5.9765e-001
tl2: 5.8757e-001
tl3: 6.0000e-001
l_cofm: 1.7902e+000
l_total: 1.9652e+000

Frequencies of modes

longpitch1: [3.0072e-001 4.1697e-001 6.6644e-001]
longpitch2: [9.7616e-001 1.2963e+000 1.6295e+000]
yaw: [5.2193e-001 1.4053e+000 2.0724e+000]
transroll1: [4.1621e-001 9.7476e-001 1.5696e+000]
transroll2: [2.3102e+000 3.9491e+000 1.5565e+001]
vertical: [1.2483e+000 4.1916e+000 1.0928e+001]

Parameters used in optimising program (opt.m) for blade design
alpha = 1.54; %shape factor
Young's Modulus = 186e9